A NEW HYBRID METHOD TO OBTAIN VISCOELASTIC
PRINCIPAL STRESS

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ABSTRACT: The basic relations in linear isotropic photoviscoelasticity have been theoretically discussed in detail. A new routine has been found to obtain the time-dependent principal stress without the measurement of isoclinics. As a test of our method, examples are given at the end of this paper.

KEY WORDS: photoviscoelasticity, Laplace transformation, boundary element method, hybrid method

I. INTRODUCTION

With the development of science and technology, more and more polymer materials are being used and this leads to the study of the stress and strain histories in the components. Because some polymer materials are transparent and birefringent, photoviscoelasticity can be used to study their mechanical properties. As a tool for visco-stress analysis, it offers a great prospect.

Our work here is restricted to linear consideration as many photoviscoelastic materials are found to follow a linear viscoelastic response over a relatively wide range of stress and time. Generally speaking, the optical and mechanical principal directions of a material are time-dependent and do not coincide with each other. Due to this reason, it is hard to translate the information from the optical measurement obtained in experiments into the actual stress distribution to be analysed. Until now most of the work is limited to certain simple cases or just concentrated on the measurement of material properties. As the isoclinics are time-dependent, it is impossible to measure all isoclinics at a specific time. It means that the normal shear stress difference method will fail. In order to overcome these difficulties, we try to find out the principal stress, instead of the stress-tensor components, in the viscoelastic body. As useful mathematical tools, the Laplace transformation and boundary element method have been used.

II. DETERMINATION OF THE FIRST STRESS INVARIANT

For any isotropic linear viscoelastic material, the relation between stress tensor and strain tensor can be expressed as follows:

\[
e_{ij}(x,y,t) = P(t) s_{ij}(x,y,0^+) + \int_0^t P(t-\tau) \frac{\partial}{\partial \tau} s_{ij}(x,y,\tau) d\tau
\]

\[
e(x,y,t) = Q(t) s(x,y,0^+) + \int_0^t Q(t-\tau) \frac{\partial}{\partial \tau} s(x,y,\tau) d\tau
\]

where \(P(t)\) and \(Q(t)\) are the creep functions, \(e_{ij}\) and \(s_{ij}\) are the deviatoric strain and stress tensors, and \(s\) and \(e\) are the hydrostatic stress and volume dilatation, respectively. Therefore, we have

\[
e = e_{ii}
\]

\[
e_{ij} = e_{ij} - e \delta_{ij}/3
\]

\[
s = \sigma_1/3
\]

\[
s_{ij} = \sigma_{ij} - s \delta_{ij}
\]

\[
\delta_{ij} = 1 \quad \text{if } i=j
\]

\[
\delta_{ij} = 0 \quad \text{if } i\neq j
\]

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The application of Laplace transformation on both sides of Eqs. (1a, b) yields
\[ e_{ij}(x_1, x_2, p) = p \bar{P}(p) \bar{e}_{ij}(x_1, x_2, p) \] (3a)
\[ e_i(x_1, x_2, p) = p \bar{Q}(p) \bar{e}_i(x_1, x_2, p) \] (3b)
where \( p \) is the Laplace parameter and \( * \rightarrow * \) means Laplace transform.

For two-dimensional viscoelasticity, we have the following equilibrium conditions and kinematic relations:
\[ \sigma_{ij}(x_1, x_2, t) = 0 \]
\[ e_{ij}(x_1, x_2, t) = \frac{[u_{i,j}(x_1, x_2, t) + u_{j,i}(x_1, x_2, t)]}{2} \]
or, with the Laplace transform,
\[ \bar{\sigma}_{ij}(x_1, x_2, p) = 0 \] (4a)
\[ \bar{e}_{ij}(x_1, x_2, p) = \frac{[u_{i,j}(x_1, x_2, p) + u_{j,i}(x_1, x_2, p)]}{2} \] (4b)

By differentiation of Eq. (4a), we have
\[ \bar{\sigma}_{ij,j}(x_1, x_2, p) = 0 \]
\[ \bar{e}_{ij,j}(x_1, x_2, p) = 0 \]
i.e.
\[ \bar{\sigma}_{12,21}(x_1, x_2, p) + \bar{\sigma}_{11,11}(x_1, x_2, p) + \bar{\sigma}_{22,22}(x_1, x_2, p) = 0 \] (5a)
and, similarly, from Eq. (4b), we obtain
\[ \bar{e}_{11,22}(x_1, x_2, p) + \bar{e}_{22,11}(x_1, x_2, p) + 2\bar{e}_{12,12}(x_1, x_2, p) = 0 \] (5b)

From Eqs. (2) and (3), we obtain
\[ \bar{e}_{11}(x_1, x_2, p) = \bar{e}_{11}(x_1, x_2, p) + \bar{e}(x_1, x_2, p)/3 = p \bar{P}(p) \bar{e}_{11}(x_1, x_2, p) + p \bar{Q}(p) \bar{e}(x_1, x_2, p)/3 \]
\[ \bar{e}_{22}(x_1, x_2, p) = \bar{e}_{22}(x_1, x_2, p) + \bar{e}(x_1, x_2, p)/3 = p \bar{P}(p) \bar{e}_{22}(x_1, x_2, p) + p \bar{Q}(p) \bar{e}(x_1, x_2, p)/3 \]
\[ \bar{e}_{12}(x_1, x_2, p) = \bar{e}_{12}(x_1, x_2, p) = p \bar{P}(p) \bar{e}_{12}(x_1, x_2, p) \]

After substitution of the above equations into Eq. (5b), the following equation is obtained
\[ p \bar{P}(p) [\bar{e}_{11,22}(x_1, x_2, p) + \bar{e}_{22,11}(x_1, x_2, p)] + p \bar{Q}(p) [\bar{e}_{12,12}(x_1, x_2, p)] = 0 \]
Eliminating \( \bar{e}_{12}(x_1, x_2, p) \) by Eq. (5a) and considering the definition of \( \bar{s} \), for two dimensional stress case, we obtain
\[ p [\bar{Q}(p) + 6\bar{P}(p)] [\bar{s}_{11}(x_1, x_2, p) + \bar{s}_{22}(x_1, x_2, p)] = 0 \] (6)

By introducing the first stress invariant \( I \) and the Laplace differentiation operator \( \nabla^2 \), we have
\[ \bar{p} \bar{A}(p) \nabla^2 \bar{I}(x_1, x_2, p) = 0 \] (7)
where \( \bar{A}(p) = \bar{Q}(p) + 6\bar{P}(p) \) depending only on the material properties
\[ I(x_1, x_2, t) = \sigma_{ii}(x_1, x_2, t) = 3s(x_1, x_2, t) \]

The inverse transformation of Eq. (7) yields
\[ \bar{A}(t) \nabla^2 \bar{I}(x_1, x_2, 0^+) + \int_{0^+}^{t} A(t-\tau) \frac{\partial}{\partial \tau} [\nabla^2 \bar{I}(x_1, x_2, \tau)] d\tau = 0 \] (8a)

The differential operator is related only to space coordinates and has nothing to do with time, and therefore Eq. (8a) can be written in the following form
\[ \nabla^2 \left[ A(t) I(x_1, x_2, 0^+) + \int_{0^+}^{t} A(t-\tau) \frac{\partial}{\partial \tau} I(x_1, x_2, \tau) d\tau \right] = 0 \] (8b)