SOME ISSUES OF TURBULENCE STATISTICS*

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ABSTRACT: The issue of dropping the random force $f_i$ and the arbitrariness of choosing the basic variable in the variational approach to turbulence closure problem, pointed out recently by the Russian scientists Bazdenkov and Kukharkin, are discussed. According to the mean-square estimation method, the random force $f_i$ should be dropped in the error expression of the LFP (Langevin-Fokker-Planck) model. However, $f_i$ is not neglected, its effect has been taken into account by the variational approach. In order to optimize the perturbation solution of the Liouville equation, the LFP model requires that the basic variable is as near to Gaussian as possible. Hence, the velocity, instead of the vorticity, should be chosen as the basic variable in the three-dimensional turbulence. Although the LFP model and the zero-order Gaussian term of PDF (probability density function) imply whiteness assumption (zero correlation time of $f_i$), the higher-order non-Gaussian terms of PDF correspond to the nonwhiteness of turbulence dynamics, the variational approach does calculate the nonwhiteness effect properly.

KEY WORDS: statistical theory of turbulence, variational method, closure problem, optimal control parameter, probability density function

I. INTRODUCTION

Recently Bazdenkov and Kukharkin[1] made an interesting study of the variational approach to the closure problem of turbulence, with particular attention to the perturbation-variation method of Qian[2]. They pointed out (p2249 of Ref.[1]) that the method of Qian "looks very attractive because of its clear physical grounds" and "succeeded in getting the Kolmogorov scaling and a very good agreement with the Kolmogorov constant value from experiments", however "this method is also not free from arbitrariness". They concluded that "an arbitrariness in some form is intrinsic in all closure methods, since all of them contain a certain approximation", and call it "the principal inevitability of arbitrariness in closure methods" or being "not self-consistent".

We have no quarrel with the opinion of Bazdenkov and Kukharkin that all closure methods contain certain approximations. Actually we have remarked[3], "Any approach to turbulence closure problem (so-called closure method) inevitably contains some approximations; of course the approximations should be reasonable and workable". In this paper, the author discusses mainly two interesting problems put forward by Bazdenkov and Kukharkin. The first problem is how to justify dropping the random force $f_i$ in the mean-square error

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expression of the LFP (Langevin-Fokker-Planck) model. The second problem is whether the choice of the basic variable is arbitrary. Bazdenkov and Kukharkin have studied both the three-dimensional and two-dimensional cases. In this paper we only discuss the three-dimensional turbulence. The two-dimensional case has some peculiar features and will be studied in another paper.

We adopt the notations of Refs.[1,2]. The Navier-Stokes equation is

$$\frac{d}{dt} X_i = -\nu_i X_i + \sum_{j,m} A_{ijm} X_j X_m$$

Let $P$ be the PDF (probability density function) of turbulence, which satisfies the Liouville equation $(\frac{\partial}{\partial t} + \hat{L})P = 0$, $\hat{L}$ is the Liouville operator, and its structure is determined by the Navier-Stokes equation (1). For stationary turbulence, the Liouville equation becomes

$$\hat{L}P = 0$$

The Liouville operator is quite complicated, we do not know how to obtain the exact solution of Eq.(2). Some approximate method has to be used. In the perturbation method used in classical and quantum mechanics, a complicated Hamiltonian $H$ is expressed as the sum of a simpler major part $H_0$ and a perturbation part $\Delta H$, i.e. $H = H_0 + \Delta H$. Similarly we let

$$\hat{L} = \hat{L}(0) + \Delta \hat{L}$$

and

$$P = P(0) + P(1) + P(2) + \ldots$$

Substituting (3) and (4) into (2), we have

$$\hat{L}(0) P(0) = 0$$

$$\hat{L}(0) P(1) = -\Delta \hat{L} P(0)$$

$$\hat{L}(0) P(2) = -\Delta \hat{L} P(1)$$

In order that the perturbation procedure (3)-(5) is reasonable and workable, the major operator $\hat{L}(0)$ is required to satisfy the following two conditions. 1) $\hat{L}(0)$ is simpler than the total operator $\hat{L}$ so that the Eqs.(5) are analytically solvable. 2) The perturbation operator $\Delta \hat{L} \equiv \hat{L} - \hat{L}(0)$ is as small as possible. However, the two conditions are contradictory, a compromise has to be made. Some concepts of the estimation and optimization methods can help us to make a compromise between these contradictory requirements. For example, the adjectives “better” and “worse”, instead of “correct” and “wrong”, are used to describe the result of a compromise. Moreover we hope that there are many candidates for the major operator $\hat{L}(0)$, so we can compare them and select the best (or optimal) one. The larger the set of candidates is, the better the result of a compromise is. In Qian’s 1983 paper, the candidates for major operator are the Fokker-Planck operator

$$\hat{L}(f) = -\sum_i \eta_i \left( \frac{\partial}{\partial X_i} X_i + \phi_i \frac{\partial^2}{\partial X_i^2} \right)$$

Here $\phi_i$ is the average energy of mode $i$ and is related to the energy spectrum of turbulence[2]. The $\eta_i$ are index parameters of the candidates for major operator, different $\eta_i$ represent different candidates, so there is an infinite number of candidates. The Fokker-Planck operator