SINGLE CRYSTAL HARDENING
— MECHANISMS AND DESCRIPTION

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ABSTRACT: Various work hardening mechanisms, on the basis of their characteristics, are classified into two categories: the pile-up hardening and the tangle hardening. Together with a discussion about the experimental investigations of work hardening, the descriptions for those two basic hardening mechanisms are suggested, respectively. Combining these descriptions, a new single crystal hardening law is proposed, which can be used to describe work hardening, particularly the cyclic hardening of a crystal grain in a polycrystalline material. Furthermore, some relevant discussion on the new single crystal hardening law is also made in this paper.

KEY WORDS: crystal plasticity, single crystal hardening, cyclic hardening

I. INTRODUCTION

A proper description of single crystal hardening (SCH) is necessary for micromechanics to be used for researches on the deformation and the failure of materials. A description of SCH was first made by Taylor[1], and since then, a continuous development has been achieved in this aspect[2–10]. With the help of the super-computer, micromechanical models have been successfully applied to analyse the mechanical characteristics of materials[8,11].

Generally speaking, if the plastic deformation of a crystal far exceeds its elastic deformation, current SCH descriptions are applicable (e.g., Ref.[1,9]). In the case of low-cycle fatigue, however, plastic deformation of a crystal is comparable with its elastic deformation and furthermore, cyclic hardening will be involved, therefore a better understanding of SCH is in need. In this paper, we shall first give a brief discussion of the SCH mechanisms relevant to the situation mentioned above. According to their characteristics, these mechanisms are classified into two categories and their relations with the SCH description are analysed, respectively. Combining the above analysis with the discussion of SCH experiments, we propose new SCH description, which is used in Ref.[12], to predict the responses of a polycrystal under non-proportional cyclic loading.

It should be pointed out that the single crystals considered in this paper are in situ, that is, with grain boundaries, and the single crystals without grain boundaries are called free crystals. In addition, only slip mechanism is taken into account for plastic deformation of a crystal, and the Schmid law[13] is supposed. Any boldfaced letter is reserved as a symbol for vector or tensor.
II. ESTABLISHMENT OF SCH LAW

2.1 A Brief Summary of Work Hardening Mechanisms

As SCH law is the mathematical description of work hardening, we begin this section with a brief discussion of work hardening mechanisms. There are various work hardening mechanisms (e.g., Ref.[14–17]). According to their characteristics, however, these mechanisms can be classified into two categories: the pile-up hardening and the tangle hardening. The detailed explanation is given in the following:

(1) The pile-up hardening brings about long-range stress fields, that is, the pile-up hardening affects moving dislocations in the form of a force, which means that the pile-up hardening possesses characteristics of the kinematic hardening with respect to a certain slip system. Grain boundaries and dispersed impurities are the typical factors leading to the pile-up hardening.

(2) The tangle hardening causes short-range stress fields, that is, the tangle hardening acts on a moving dislocation as a constraint, which means that the tangle hardening has the characteristics of the isotropic hardening for a certain slip system. The tangle of dislocations is the main factor resulting in the tangle hardening.

(3) A moving dislocation is not only affected by the stress fields from the pile-up hardening, but also constrained by dislocation structures of the tangle hardening. Since these two hardenings processes exert their influences on a moving dislocation independently, the total work hardening is simply a summation of the contributions from both the pile-up hardening and the tangle hardening.

2.2 Description of SCH

The classification and relevant discussion of hardening mechanisms provide a starting point and some basis for the description of a SCH law, which will be applied to the establishment of a SCH law as follows. A general form of SCH law given by Hill[18] is

\[
\dot{\tau}^i = \sum_j h^{ij} \dot{\gamma}^j
\]

where \(\dot{\tau}^i\) is the rate of yield stress of the \(i\)th slip system, and \(\dot{\gamma}^j\) is the shear rate (engineering definition) of the \(j\)th slip system. \(h^{ij}\) are called the instantaneous hardening coefficients which depend, in general, on the previous deformation history. By the way, slip systems with identical slip plane and opposite direction of slip are treated as different ones in this paper.

2.2.1 \(h^{ij}\) corresponding to the pile-up hardening

In the light of an analysis of the Orowan’s dispersion hardening, Weng[6] proposed the following hardening coefficients

\[
h^{ij} = \beta^i \beta^j \tilde{h}^j
\]

(no sum over \(j\))

where \(\beta^i = n^i \otimes s^i\) (\(\otimes\) is used for a dyad, and the contracted product of two second-order tensors \(A\) and \(B\) is simply denoted by \(AB\)), \(n^i\) is the normal to the plane of the \(i\)th slip system, and \(s^i\) is its slip direction. \(\tilde{h}^j\) is the hardening rate of the \(j\)th slip system caused by the pile-ups on itself. It is obvious that the hardening coefficients given by (2) are in accordance with the properties of the pile-up hardening.