A LATTICE BOLTZMANN MODEL FOR COMPRESSIBLE PERFECT GAS*

Hu Shouxin (胡守信) Yan Guangwu (阎广武) Shi Weiping (施卫平)
(Mechanics Section, Department of Mathematics, Jilin University, Changchun 130023, China)

ABSTRACT: A new lattice Boltzmann model for compressible perfect gas is proposed. The numerical example shows that it can be used to simulate shock wave and contact discontinuity. The results are comparable with those obtained by traditional methods. The ratio of specific heats $\gamma$ may be chosen according to the requirement of problems.

KEY WORDS: lattice Boltzmann method, compressible, perfect gas, general case of the ratio of specific heats, two-speed three-energy-level model

1 INTRODUCTION

Ten years ago, the Lattice Gas Automata (LGA) model was presented by Frisch, Hasslacher and Pomeau\cite{1} to simulate the two-dimensional fluid flows. It has attracted much attention. Not very long since then, its rapid developments were obstructed by two fundamental difficult problems, that is, the density dependence of convection coefficient $g(\rho)$ and the nonphysical velocity dependence of pressure. In 1992, these difficulties were removed in the lattice Boltzmann Method (LBM) developed by Qian and d’Humières et al.\cite{2}, Chen HD and Chen SY et al.\cite{3}, Benzi and Succi et al.\cite{4}. The LBM provides a 7-bit isothermal, incompressible model of fluid problems obeying the Navier-Stokes equations. Although it has showed strong spatial gradient phenomena in some examples of the wave propagation with initial large perturbation \cite{5}, generally speaking, it can only be used in incompressible problems with small velocity limit.

Recently, many studies of the LBM has been concentrated on the compressible flows. Alexander and Chen et al.\cite{6} formulated an isothermal LBM model which can include shocks. It has a selectable sound speed. This feature allows one to simulate compressible fluid flows with high Mach numbers. In paper \cite{7}, Qian and Orszag studied the nonlinear deviation of the LBGK model in compressible regimes, and presented a numerical simulation of a shock profile. Qian and Orszag\cite{8} also simulated a regime of weak compressibility at high Reynolds numbers. The LBM scheme was used to study the Kolmogorov flows. Ancona\cite{9} developed a class of “fully-Lagrangian” methods, and provided new perspective on the relationship

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between LBM and finite difference techniques. Two simpler areas of application of this scheme were Burger’s equations and one dimensional gas dynamics. However, the LBM model for computing strong discontinuity in gas dynamics has not been carried out yet.

In Section 2 of this paper, we describe a new LBM model for two-dimensional compressible perfect gas. In the sense of first order of accuracy the Euler equations can be recovered from the model, and the ratio of specific heats \( \gamma \) may be chosen as a parameter. The problem of parameter choice is discussed in Section 3, while in Section 4, a numerical example, the Sod’s problem of one dimensional shock tube, is given. It shows the formation of shock wave and contact discontinuity.

2 LATTICE BOLTZMANN MODEL FOR COMPRESSIBLE PERFECT GAS

The model presented in this paper is based on the FHP-III 7-bit model and the available LBM model, but we assume that the particles moving along every link are separated into two kinds, type \( A (\alpha = 1, \cdots, 6) \) and type \( B (\alpha = 7, \cdots, 12) \), which are on two energy levels \( \varepsilon_A \) and \( \varepsilon_B (\varepsilon_A > \varepsilon_B > 0) \), respectively, and the rest particles \( (\alpha = 0) \) are on another energy level \( \varepsilon_D > 0 \). So it is actually a 13-bit model.

The single particle distributions in the “shooting-in” and “shooting out” state at site \( r \) and time \( t \) are respectively denoted by \( f_\alpha (r, t) \) and \( f'_\alpha (r, t) \). We define the mass, momentum and total energy per site as follows

\[
\rho = \sum_\alpha f_\alpha \\
\rho u_i = \sum_\alpha f_\alpha e_{\alpha i} \quad (i = 1, 2) \\
\frac{1}{2} \rho u^2 + \rho E = \sum_\alpha f_\alpha \varepsilon_\alpha \quad (\varepsilon_\alpha = \varepsilon_A, \varepsilon_B, \varepsilon_D)
\]

where \( e_\alpha \) is the velocity vector of the moving particles along the link in the direction \( \alpha \), \( |e_\alpha| = c \quad (\alpha = 1, \cdots, 12) \), and \( E \) is the internal energy per unit mass.

The evolution of the system from time \( t \) to \( t + \Delta t \) is still divided into two steps: collision and streaming, and described by the following BGK-type Lattice Boltzmann Equation (LBE)

\[
f_\alpha (r + e_\alpha \Delta t, t + \Delta t) = f'_\alpha (r, t) = f_\alpha (r, t) - \frac{1}{\tau} (f_\alpha (r, t) - f^\text{eq}_\alpha (r, t)) \quad (\alpha = 0, 1, \cdots, 12)
\]

where \( \tau \) is the single-relaxation time, \( f^\text{eq}_\alpha \) is the local equilibrium distribution. We assume that \( f^\text{eq}_\alpha \) has the same expression as that in the available 7-bit LBM\(^{[2-4]}\)

\[
\begin{align*}
    f^\text{eq}_0 & = D_0 \rho + D_3 \rho u^2 \\
    f^\text{eq}_\alpha & = A_0 \rho + A_1 \rho u_i e_{\alpha i} + A_2 \rho u_i u_j e_{\alpha i} e_{\alpha j} + A_3 \rho u^2 & (\alpha = 1, \cdots, 6) \\
    f^\text{eq}_\alpha & = B_0 \rho + B_1 \rho u_i e_{\alpha i} + B_2 \rho u_i u_j e_{\alpha i} e_{\alpha j} + B_3 \rho u^2 & (\alpha = 7, \cdots, 12) \\
    f^\text{eq}_0 & = D_0 \rho + D_3 \rho u^2
\end{align*}
\]

We adopt the idea proposed by Yan\(^{[10]}\) that besides the conservation conditions of mass, momentum and energy: