ON THE FORMATION OF VORTEX RINGS AND PAIRS*

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ABSTRACT: The axisymmetric vortex sheet model developed by Nitsche & Krasny (1994) has been extended to study the formation of vortex rings (pairs) at the edge of circular (2D) tube and opening. Computations based on this model are in good agreement with the experiments (Didden (1979) for circular tube and Auerbach (1987) for 2D tube and opening). Using this new model, evidences are provided to show that the main failure of the similarity theory (the false prediction of axial trajectory of vortex ring) is due to its ignorance of the self-induced ring velocity (mutual induction for vortex pair). We further reason why the similarity theory succeeds in its prediction of radial movement of vortex ring. The effects of various parameters such as turning angle $\alpha$ and piston speed $U_p(t)$ on the formation of vortex ring are investigated. Numerical result shows that turning angle $\alpha$ has no effect on circulation shed $\Gamma'$. We also discuss Glezer (1988)'s summary on the influence of $U_p$ upon the shedding circulation, and finally give the variation of core distribution of vortex ring with $\alpha$ and $U_p(t)$.

KEY WORDS: formation of vortex ring (pair), vortex method, the similarity theory

1 INTRODUCTION

Vortex ring, a kind of fairly robust state of concentrate vortices, easily isolated from end-effects than other configurations and simple to generate, is suitable for both theoretic and experimental analysis. It also plays an important role in the dynamics of round jet as coherent structure.

Vortex rings is mostly often formed by pushing fluid through a nozzle with turning angle $\alpha$ (see Fig.1, vortex ring generator), tube for $\alpha = 0^\circ$ and opening for $\alpha = 90^\circ$. A general vortex ring generator has the following parameters: turning angle $\alpha$, stroke length $L_M$, diameter $D_M$, piston speed $U_p$.

Fig.1 The schematic view of vortex ring generator

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and kinematic viscosity of the fluid $\nu$. The dimensionless parameters are $L_M/D_M$, $Re = \overline{U}_p D_M/\nu$ and $P = (1/T) \int_0^T (U_p(t)/\overline{U}_p)^2 dt$, together with $\alpha$, by which the formation process of a vortex ring is determined (Maxworthy 1977, Saffman 1978 and Glezer 1988)\cite{1-3]. The characteristics of vortex ring in which we are interested are trajectory of the core center $(x_c/D_M, r_c/D_M)$, axial moving speed $U_T/\overline{U}_p$, ring diameter $D/D_M$, core diameter $d/D_M$, circulation shed $\Gamma/\overline{U}_p D_M$ and core distribution $\Gamma_d(\rho)$, where $(\rho, \theta)$ is the coordinate system with core center as original point.

Since the development of vortex ring during its formation process plays a key role in the later characteristics of the ring, theoretical models have been set up (see the review article by Shariff & Leonard 1992\cite{4}) to reveal its underlying physics and dependence upon the generating parameters. By simplifying the vortex ring generator with a turning flow around 2D semi-infinite sharp edge with negative $x$-axis along the coming flow, the similarity theory gives out the complex potential as $\Phi = at^m z^n$, where $n = \pi/(2\pi - \alpha)$ and $U_p(t) = Bt^m$. Because $a$ is the only dimensional parameter, the vortex sheet separating at the edge will rollup to form a spiral in a self-similar way. Dimensional analysis (Saffman 1978)\cite{2} shows

$$d = c_1(m, n)a^{1/(2-n)t^{(m+1)/(2-n)}}$$

(1)

$$\Gamma = c_2(m, n)a^{2/(2-n)t^{(m+1)/(2-n)-1}}$$

If $\alpha = 0^\circ$, $U_p$ is constant, then we have $d \propto t^{2/3}$, $\Gamma \propto t^{1/3}$ and $D/D_M = 1 + k_5 (L/D_M)^{2/3}$, where $k_5 = 0.32$ (Pullin 1979)\cite{5}. Setting $\Gamma_0 = \int_0^L U_p(L')dL'$, we have $\Gamma/\Gamma_0 \propto (L/D_M)^{-2/3}$. Pullin (1978)\cite{6} and Auerbach (1987)\cite{7} further gave out the trajectory of vortex ring as $x_c \propto t^{2/3}$, $r_c \propto t^{2/3}$, which is a straight line and $x_c < 0$.

The formation of vortex ring at a tube is carefully studied by Didden (1979)\cite{8}, who found that although the theoretical prediction $r_c \propto t^{2/3}$ is coincident with the experiment, $x_c \propto t^{2/3}$ and $\Gamma \propto t^{1/3}$ are incorrect compared with the experimental result $x_c \propto t^{3/2}$ and $\Gamma \propto t$ after $t = 0.6$. Didden attributes this discrepancy to the theory's neglect of the self-induced ring velocity, but he has not provided any evidence. Through experimental investigation of vortex pairs formation at the edge of both 2D tube and opening, Auerbach (1987)\cite{7} observed the same behaviour of $x_c$ in 2D flow. After excluding the effect of axisymmetry, viscous diffusion, distant boundary influence and shear layer thickness, Auerbach (1987)\cite{7} concluded that the discrepancy between the similarity theory and experiment is caused by the theory's ignorance of either secondary vorticity generated on the outer tube wall or viscous entrainment by the developing jet. Nitsche & Krasny (1994)\cite{9} restudied this problem using a new vortex sheet model, based on the computation which is in close agreement with the experiment of Didden (1979)\cite{8}, they claimed that although the neglect of self-induced ring velocity is a factor for the failure of similarity theory, the geometry of the solid boundary is responsible for the discrepancy in both axisymmetric ring and 2D pair. Because in both cases, the starting flow contains a downstream component which is absent in the similarity theory.

On the other hand, the similarity theory is quite accurate in the characteristics of vortex ring formation such as the radial trajectory $r_c$ of vortex ring and the $x_c$ and $\Gamma$ at small times (Nitsche 1995)\cite{10}. With the help of Nitsche & Krasny's model\cite{9}, we hope to provide strong evidence to show why the similarity theory fails in its prediction of $x_c$ and