A LAGRANGIAN LATTICE BOLTZMANN METHOD FOR EULER EQUATIONS

Yan Guangwu

(Laboratory for Nonlinear Mechanics of Continuous Media, Institute of Mechanics, Chinese Academy of Sciences, Beijing 100080, China)

(Department of Mathematics, Jilin University, Changchun 130023, China)

ABSTRACT: A Lagrangian lattice Boltzmann method for solving Euler equations is proposed. The key step in formulating this method is the introduction of the displacement distribution function. The equilibrium distribution function consists of macroscopic Lagrangian variables at time steps $n$ and $n + 1$. It is different from the standard lattice Boltzmann method. In this method the element, instead of each particle, is required to satisfy the basic law. The element is considered as one large particle, which results in simpler version than the corresponding Eulerian one, because the advection term disappears here. Our numerical examples successfully reproduce the classical results.

KEY WORDS: Lagrangian lattice Boltzmann method, displacement distribution functions, Lagrangian coordinates, Euler equations

1 INTRODUCTION

In the past years, the lattice Boltzmann equation (LBE) has been developed as a computational fluid dynamics (CFD) method. This method originated from a Boolean model known as the Lattice Gas Automata (LGA) which simulates the Navier-Stokes equations on a regular lattice. As an important progress, the simple collision model of BGK was applied to the lattice Boltzmann equation, yielding the lattice BGK model. However, this method is limited in a range of low Mach number as an image gas. In recent years, a series of lattice Boltzmann models for compressible perfect gas were proposed. In Ref. [5] and [6] we used flux conditions and energy or entropy level to remove the inconsistency of the moments, but the accuracy of the model is not good enough.

As is well known, Lagrangian forms are better suited to trace discontinuity because they have no advection terms. M.G. Ancona defined a "fully-Lagrangian" lattice Boltzmann method for the conservational system, a key step in the approach to use Lagrangian variables, in which the lattice boltzmann equation becomes Lagrangian lattice Boltzmann equation (LLBE). Our paper also uses Lagrangian lattice Boltzmann equation, but has three aspects differing from Ancona's method: (i) we use displacement distributions instead of velocity distribution, (ii) macroscopic variables are mass, displacement and pressure force,
instead of density, momentum and energy, (iii) the equilibrium distribution function consists of macroscopic variables at time steps $n$ and $n + 1$. We discreted flow fields into elements in which only mass is conserved, this element is a "large" particle with pressure force, therefore, it has unique conservational variables mass, thus the equilibrium distribution becomes simpler than the corresponding Eulerian versions. In section 2, we introduce the Lagrangian lattice Boltzmann equation and displacement distribution function, and then give the definition of macroscopic variables, in section 3, we give three numerical examples. The accuracy and stability will be discussed in another paper.

2 LAGRANGIAN LATTICE BOLTZMANN EQUATION

The Lagrangian coordinates are used in the present study. At time $t = 0$, we divided flow fields into square elements, the nodes are $(i, j), (i + 1, j), (i + 1, j + 1), (i, j + 1)$ surrounding an element $(i, j)$, define $(i, j)$ as Lagrangian coordinates. Vector $e_{\alpha}$ is the displacement in direction $c_{\alpha} (\alpha = 1, \cdots, b)$. We define $f_{\alpha}$ as the element $(i, j)$ displacement distribution along the $\alpha$th direction at time $t$, thus mass is the conservational variables, see Fig.1.

$$\sum_{\alpha} f_{\alpha} = m = \sum_{\alpha} f_{\alpha}^{eq} \quad (1)$$

the displacement of element $(i, j)$ is defined as

$$\sum_{\alpha} e_{\alpha\sigma} f_{\alpha} = mX_{\sigma} \quad (2)$$

![Fig.1](a) Lagrangian coordinate (b) displacement

The Lagrangian lattice Boltzmann equation is expressed as

$$f_{\alpha}^{n+1} = f_{\alpha}^{n} - \frac{1}{\tau}[f_{\alpha}^{n} - f_{\alpha}^{eq,n}] = g_{\alpha}^{n} \quad (3)$$

where $f_{\alpha}^{eq}$ is the local equilibrium distribution function, $\tau$ is the relaxation factor, $g_{\alpha}$ is the collision term, $\Delta t$ is the time step, $n$ refers to the time.

The collision term is required to satisfy

$$\sum_{\alpha} g_{\alpha}^{n} = 0 \quad (4)$$

$$\sum_{\alpha} g_{\alpha}^{n} e_{\alpha\sigma} = m\bar{u}_{\sigma}^{n} \quad (5)$$

$$\sum_{\alpha} e_{\alpha\sigma} g_{\alpha}^{n+1} - g_{\alpha}^{n} = \bar{N}_{\sigma}^{n} \quad (6)$$