VOID GROWTH AND CAVITATION IN NONLINEAR VISCOELASTIC SOLIDS*

ZHANG Yun (张 贤)† HUANG Zhuping (黄筑平)
(Department of Mechanics and Engineering Science, Peking University, Beijing 100871, China)

ABSTRACT: This paper discusses the growth of a pre-existing void in a nonlinear viscoelastic material subjected to remote hydrostatic tensions with different loading rates. The constitutive relation of this viscoelastic material is the one recently proposed by the present authors, which may be considered as a generalization of the non-Gaussian statistical theory in rubber elasticity. As the first order approximation, the above constitutive relation can be reduced to the "neo-Hookean" type viscoelastic one. Investigations of the influences of the material viscosity and the loading rate on the void growth, or on the cavitation are carried out. It is found that: (1) for generalized “inverse Langevin approximation” nonlinear viscoelastic materials, the cavitation limit does not exist, but there is a certain (remote) stress level at which the void will grow rapidly; (2) for generalized “Gaussian statistics” (neo-Hookean type) viscoelastic materials, the cavitation limit exists, and is an increasing function of the loading rate. The present discussions may be of importance in understanding the material failure process under high triaxial stress.

KEY WORDS: void growth, cavitation, viscoelastic, non-Gaussian statistical theory, inverse Langevin approximation

1 INTRODUCTION

The study of void nucleation and growth in solids is a subject which has attracted a great deal of interest and has been the research field of many investigators since the 1960's. For example, Rice and Tracey[1] and Budiansky et al.[2] studied the growth law of a spherical void in rigid perfectly plastic and power-law viscous materials, respectively. By using a unit cell of a rigid perfectly plastic medium, Gurson[3] derived the upper bound expressions of the yield function for porous materials. A comprehensive discussion on the void growth may be found in review articles by Tvergaard[4] and Huang et al.[5].

On the other hand, in contrast to void growth in a rigid plastic or in a nonlinear viscous material, the sudden void formation (or cavitation) may take place under high triaxial stress in nonlinear elastic or elastic-plastic materials. This phenomenon has been observed experimentally, for example, in vulcanized rubbers by Gent and Lindey[6], in lead wires by Ashby et al.[7], in ductile fracture of metal sheets by Dalgleish et al.[8], and has been subsequently studied theoretically by many researchers (e.g. Ball[9], Huang et al.[10], Horgan[11], Hou H-S[12], Horgan and Polignone[13]). So called cavitation under static loading conditions is actually a bifurcation problem. Under the high stress state, the void growth may occur instantaneously, with the energy available from the field surrounding the void being enough to drive its continuing expansion. Unlike the normal void growth which occurs directly in proportion to the deformation imposed on the material, the void growth rate in the cavitation will be extremely high.

In this paper, the growth of a pre-existing void in a nonlinear viscoelastic medium under remote hydrostatic tensions with different loading rates is studied. The constitutive relation of this viscoelastic medium

* The project supported by the National Natural Science Foundation of China (10032010)
† E-mail: zhang.yun@water.pku.edu.cn
is the one proposed by the present authors, which may be considered as a generalization of the non-Gaussian statistical theory in rubber elasticity. Emphases will be placed on the influences of the material viscosity and loading rate on the void growth, and/or on the cavitation. It is found that: (1) for generalized "inverse Langevin approximation" nonlinear materials, the cavitation limit does not exist, but at a certain stress level, the void will grow rapidly; (2) for generalized "neo-Hookean" materials, the cavitation limit exists and is an increasing function of the loading rate. The above phenomena may be of significance in the material failure process.

2 A CONSTITUTIVE MODEL OF A VISCOELASTIC MATERIAL AT FINITE DEFORMATION

Based on the molecular network model in solid polymers, an internal-variable theory of viscoelastic constitutive relations for compressible materials has been proposed by Huang et al.\cite{14}. In order that this theory might be applied to incompressible materials, a necessary modification has to be made. For an isothermal process, the modified constitutive relations can be written as

\[ T = -p U^{-1} + \rho_0 \frac{\partial \Psi}{\partial E} \]  
\[ A^\alpha = -\rho_0 \frac{\partial \Psi}{\partial \xi_\alpha} \]  
\[ \eta_\alpha \dot{\xi}_\alpha = A^\alpha \]  
\[ \alpha = 1, 2, \cdots \]  
\[ \text{no summation over } \alpha \]  

(1a) (1b) (2)

In above equations, \( U \) is the right stretch tensor, \( E \) is the (Lagrangian type) engineering strain, \( \xi_\alpha \) is a second rank symmetric tensor, and is called the \( \alpha \)th internal-variable. \( \Psi \) is the specific Helmholtz free energy, \( T \) and \( A^\alpha \) are the stresses conjugate to \( E \) and \( \xi_\alpha \), respectively, \( \rho_0 \) is the mass density at the reference configuration, \( \eta_\alpha \) is the \( \alpha \)th viscosity coefficient, \( p \) is the indeterminate hydrostatic pressure. In the following, we only consider the deformation in which the eigenvectors of \( E \) do not change. If the material is initially isotropic, the free energy in Eq.(1a) can be expressed as the function of three principal invariants of \( E \) and \( \xi_\alpha \), or as the function of the principal stretches \( \lambda_1 - \xi_{11}, \lambda_2 - \xi_{22} \) and \( \lambda_3 - \xi_{33} \). For the non-Gaussian statistical theory described by the inverse Langevin function \( \ell^{-1}(\beta) \), where \( \ell(\beta) = \coth \beta - \frac{1}{\beta} \), Eq.(1a) can be written alternatively as

\[ \sigma_i = -p + \rho_0 \lambda_i \frac{\psi_i^{(\alpha)}}{\alpha} (\lambda_1 - \xi_{11}, \lambda_2 - \xi_{22}, \lambda_3 - \xi_{33}) \]  
\[ i = 1, 2, 3; \text{ no summation over } i \]  

(3)

where

\[ \psi_i^{(\alpha)} = \frac{\mu_\alpha}{4\pi} \int_0^\pi d\varphi \int_0^{2\pi} \sqrt{m_\alpha} \ell^{-1} \left( \frac{\lambda_\alpha}{\sqrt{m_\alpha}} \right) \]  
\[ \frac{\partial^2}{\partial \xi_{\alpha \alpha}^2} \sin \varphi d\omega - \frac{\mu_\alpha \sqrt{m_\alpha}}{3} \ell^{-1} \left( \frac{1}{\sqrt{m_\alpha}} \right) \frac{1}{\ell} (\lambda_i - \xi_{\alpha \alpha}) \]  

(4)

In the above equation, \( \sigma_i \) and \( \xi_{\alpha \alpha} \) \( (i = 1, 2, 3) \) are the principal components of the Cauchy stress and the \( \alpha \)th internal variable, respectively. \( \rho_0 \mu_\alpha \) is the \( \alpha \)th shear modulus, \( m_\alpha \) is the number of links in the \( \alpha \)th molecular chain. \( \lambda_\alpha = \left[ \sum_{i=1}^3 \lambda_i^2 (\lambda_i - \xi_{i1})^2 \right]^{1/2} \), and

\[ l_{01} = \sin \varphi \cos \omega \]  
\[ l_{02} = \sin \varphi \cos \omega \]  
\[ l_{03} = \cos \varphi \]  

(5)

3 VOID GROWTH IN A VISCOELASTIC MEDIUM

Now consider a pre-existing spherical void with radius \( R_0 \) embedded in an infinite viscoelastic medium, which is subjected to a remote hydrostatic tension: \( \sigma_r(\infty) = a \) with constant loading rate \( a \). Suppose that the deformation is spherically symmetric, and can be expressed in spherical polar coordinates as

\[ r = r(R) > 0 \]  
\[ \theta = \Theta \]  
\[ \varphi = \Phi \]  

(6)

where \( R \) and \( r \) are the distances to a material point from the void center in the undeformed and the deformed configurations, respectively. Hence the principal stretches are

\[ \lambda_1 = \frac{dr}{dR} = r' \]  
\[ \lambda_2 = \lambda_3 = \frac{r(R)}{R} \]  

(7)

The constitutive relations will be described by Eqs.(3) and (4). In view of the incompressibility of the material, we have

\[ v(R, R_0) = \frac{r}{R} = \left( 1 + \frac{r_0^3 - R_0^3}{R^3} \right)^{1/3} \]  

(8)

where \( r_0 \) is the void radius after deformation.

In the absence of body force, the equilibrium equation becomes

\[ \frac{d\sigma_r}{dr} + 2r' (\sigma_r - \sigma_\theta) = 0 \]  

(9)