MATHEMATICAL MODELING OF CIRCULATION AND ITS VERIFICATION

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ABSTRACT: In this study, the mechanism of circulation generation in channel with step expansion is investigated. The circulation results from the balance between inertia force and bed-induced shear stress. Accordingly, the convective inertia term and the bottom resistance in the momentum equation are essential to the simulation of circulation. By means of a corresponding 2-D model, the velocity fields of unsteady circulating are computed. A comparison of the results with the velocity fields measured data is made and the agreement is satisfactory.

KEY WORDS: mathematical simulation, circulation, channel with step expansion.

I. INTRODUCTION

The modeling of circulation has practical importance in the prediction of spreading of pollution and heat, and transport of sediment in steady and unsteady flows. Circulations can often influence the morphological formation processes of coast lines and downstream side of hydraulic structures. Circulations can also make navigation conditions unfavorable. In many practical problems in hydraulic, environmental and coastal engineering, we need to investigate the fundamental mechanism of the formation of circulation.

In the past several decades, circulation has been investigated mainly with physical-hydraulic models. But in recent years, high-speed computers have become widely used and computational techniques have developed rapidly. A number of complicated hydraulic phenomena can be effectively analyzed by numerical simulations. For example, shallow-water circulation could be treated by two-dimensional nearly-horizontal flow models. In the 1960’s and 1970’s, some such mathematical models were developed and applied to the study of estuaries and coastal hydrodynamics. Later, some authors specifically dealt with the circulation problems. The key problem was how to use a two-dimensional model to simulate circulation. Kuipers and Vrengdenhil extended Leedertse’s (1967) model to the treatment of secondary flow and they concluded that effective shear stresses had to be introduced in such a model. According to Kuipers and Vrengdentil, the convective term plays an important role in simulating the circulation. If wind stress is neglected, vorticity can be created either by the convection in converging and diverging flow or by the effective shear stress. Flokstra meticulously analyzed the relevant physical mechanism of the generation of circulation. According to Flokstra’s theory of vorticity balance, the generation of circulation is theoretically impossible if the effective shear stress is neglected. Abbott and Rasmussen pointed out that circulation cannot appear, at least when using a difference approximation with reasonable accuracy, if either the inertia term or the

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bottom resistance is neglected. Abbott and Rasmussen also concluded that the "pseudo-circulations" caused by the truncation errors of first order difference schemes, would occur in two-dimensional depth-averaged models.

In this paper, based upon the analysis of the physical processes of a rapid expanding flow and the equation of vorticity transport, the generation of circulation will be investigated. As a result, we reach the conclusion that it is necessary to consider both nonlinear convective term and the bottom resistance term in the momentum equation for simulating a circulation. The comparison of a hydraulic physical model with the mathematical model confirms this conclusion, which agrees well with the results in [3].

II. BASIC EQUATIONS OF THE TWO-DIMENSIONAL NEAR-HORIZONTAL FLOW

The equations of mean motion for a turbulent flow are as follows: Continuity equation.

\[ \frac{\partial \rho}{\partial t} + \frac{\partial (\rho \bar{u}_i)}{\partial x_i} = 0 \]  

(1)

Momentum equations:

\[ \frac{\partial \bar{u}_i}{\partial t} + u_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu \frac{\partial \bar{u}_i}{\partial x_j} - u'_i u'_j \right) \]

(2)

Where

\( \bar{u}_i \) — time-average velocity
\( p \) — pressure
\( \rho \) — density
\( \nu \) — kinematic coefficient of viscosity
\( u'_i \) — fluctuating velocity

For a long wave motion, because of the small curvature of the surface profile, the vertical components of velocity and the acceleration can be neglected. For incompressible flow, the equations of two-dimensional depth-averaged flow can be obtained by integrating Eqs. (1) and (2) from bottom to surface and using the kinematic boundary conditions at the surface and the bottom. By applying the Liebnitz' rule, the equations for the two-dimensional near-horizontal flow are developed as follows: Continuity equation

\[ \frac{\partial h}{\partial t} + \frac{\partial}{\partial x} (h V) + \frac{\partial}{\partial y} (h V) = 0 \]

(3)