EXACT SOLUTIONS FOR MAGNETOHYDRODYNAMIC FLOW IN A ROTATING FLUID

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ABSTRACT: An analytical solution is obtained for the flow due to solid-body rotations of an oscillating porous disk and of a fluid at infinity. Neglecting the induced magnetic field, the effects of the transversely applied magnetic field on the flow are studied. Further, the flow confined between two disks is also discussed. It is found that an infinite number of solutions exist for the flow confined between two disks.

KEY WORDS: rotating fluid, porous disk, double disks, magnetohydrodynamic flow

1 INTRODUCTION

In the past few years there has been a considerable interest in rotating magnetohydrodynamic viscous fluid flows due to possible applications to geophysical and astrophysical problems. It is generally admitted that the force due to the earth's rotation has a strong effect on the hydromagnetic flow in the earth's liquid core. The motion in the earth's core is somehow responsible for the main geomagnetic field. It is well known that a number of astronomical bodies (e.g. the Sun, Earth, Jupiter, magnetic stars, pulsars) possess fluid interiors and (at least surface) magnetic fields. Changes in the rotation rate of such objects suggest the possible importance of hydromagnetic spin-up. This problem of spin-up in magnetohydrodynamic rotating fluids has been discussed under varied conditions by many researchers notably Gilman and Benton[1], Benton and Loper[2], Chawala[3] et al. In all these analyses, the effects of magnetic field and rotation are considered. Further, Gupta[4] obtained an exact solution of the steady three-dimensional Navier-Stokes equations for the flow past a plate with uniform suction in a rotating coordinate system. He has discussed the structure of the steady velocity field and the associated boundary layer on the porous plate. Further, he has argued that rotation is entirely responsible for the existence of a solution for the case of blowing. Soundalgekar and Pop[5] examined hydromagnetic flow in a rotating fluid past an infinite porous wall. Murthy and Ram[6] extended this analysis to study the behavior of magnetohydrodynamic Ekman layer on a flat plate, subjected to suction and blowing. Debnath[7] found the solutions for the Stokes and Ekman problems in magnetohydrodynamics. He analyzed some qualitative informations regarding (i) the structures of the boundary layers and (ii) the propagation of waves and their interaction with
the boundary layers. In another paper Debnath[8] discussed the structures of Ekman and Hartmann boundary layers with several special limiting cases. More recently, Turbatu et al.[9] presented the flow of an oscillating plate with the combination of superimposed blowing or suction and increasing or decreasing velocity amplitude.

Keeping in mind the importance of the effects of rotation and electromagnetic on the hydromagnetic flow and their applications in cosmical fluid dynamics and solar physics, the simultaneous influence of these agents may be interesting. Thus, the objective of the present study is three fold. Firstly, to investigate the unsteady analysis of hydromagnetic flow in a semi-infinite expanse of an electrically conducting rotating viscous fluid bounded by an infinite non-conducting porous oscillating disk; with the combination of superimposed suction or blowing and decreasing or increasing velocity amplitude in the presence of a transverse uniform magnetic field. Secondly, to discuss the flow due to oscillating porous disk and a fluid at infinity rotating about an axis passing through \((x_1, y_1)\) parallel to the \(z\)-axis. Thirdly, to examine the hydromagnetic flow between two disks one of which is oscillating \((at \ z = 0)\) and the other \((at \ z = d)\) rotates about an axis passing through \((x_1, y_1)\). It is found that for the case of two disks an infinite set of solutions is possible. In these studies we found that similar to Turbatu et al.[9] results a combination of suction or blowing and decreasing or increasing velocity amplitude of the oscillating disk is possible.

2 MATHEMATICAL FORMULATION

We consider the unsteady boundary layer flow induced in the semi-infinite expanse of an electrically conducting viscous fluid bounded by an infinite non-conducting porous disk at \(z = 0\) subjected to uniform suction or blowing in the presence of a transverse uniform magnetic field \(B_0\) normal to the disk. The fluid as well as the disk is in a state of solid body rotation with constant angular velocity \(\Omega\) about the \(z\)-axis normal to the disk and additionally, non-torsional oscillations of frequency \(n\) are imposed on the disk in its own plane at time \(t > 0\). Now making reference to Debnath[7] and eliminating \(\dot{\rho} = \rho - \rho \Omega^2(x^2 + y^2)/2\) the boundary layer equation is given by

\[
\frac{\partial^2 W}{\partial z \partial t} - W_0 \frac{\partial^2 W}{\partial z^2} + (2i\Omega + N) \frac{\partial W}{\partial z} = \nu \frac{\partial^3 W}{\partial z^3} \quad z > 0 \quad (1)
\]

where

\[
W = u + iv \quad N = \frac{\sigma B_0^2}{\rho}
\]

and \(\rho\) is the density, \(\sigma\) the electrical conductivity, \(u\) and \(v\) are the \(x\)- and \(y\)-components of velocity, respectively.

In view of the imposed oscillations on the disk, Eq.(1) has to be solved subject to the boundary conditions

\[
W = ae^{(\beta + in)t} + be^{(\beta - in)t} \quad \text{on} \quad z = 0 \quad t > 0 \quad (2)
\]

where \(a, b\) are complex constants, \(n(> 0)\) is the fixed frequency of oscillations. \(\beta = \text{constant} \neq 0\) with simultaneous suction or blowing \(W_0 = \text{constant} \neq 0\). The boundary condition corresponding to the fluid at infinity rotating about an axis passing through \((x_1, y_1)\) parallel to \(z\)-axis is given by

\[
W = x_1 + iy_1 \quad \text{at} \quad z \to \infty \quad t > 0 \quad (3)
\]