DETERMINATION OF THE DYNAMIC STRESS INTENSITY FACTORS, $K_{I}^d$ AND $K_{II}^d$, FOR A MIXED-MODE PROPAGATING CRACK

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ABSTRACT: In this paper, the dynamic propagation problem of a mixed-mode crack was studied by means of the experimental method of caustics. The initial curve and caustic equations were derived under the mixed-mode dynamic condition. A multi-point measurement method for determining the dynamic stress intensity factors, $K_{I}^d$ and $K_{II}^d$, and the position of the crack tip was developed. Several other methods were adopted to check this method, and showed that it has a good precision. Finally, the dynamic propagating process of a mixed-mode crack in a three-point bending beam specimen was investigated with our method.

KEY WORDS: caustic method, stress intensity factor, dynamic fracture.

I. INTRODUCTION

In a brittle material, a propagating crack will often depart from its original trajectory and curve or split into two or more branches. For dealing with this kind of problems, an experimental method which can determine the dynamic stress intensity factors, $K_{I}^d$ and $K_{II}^d$, effectively and conveniently, is required. The caustic technique is an optical method in the experimental stress analysis, and has been widely used in the study of singular stress field in the engineering problems. But for dynamic case, it was mainly limited to the mode I (opening mode) propagating crack. Theocaris and Papadopoulos proposed a caustic method to measure the mixed-mode dynamic stress intensity factors. They utilized the sizes and the relative position of the two caustic curves formed by the light rays reflected from the front and rear surfaces of the specimen, respectively. Their method, therefore, can only be applied to transparent materials. Now the question is: whether or not we can determine the dynamic stress intensity factors of a mixed-mode propagating crack by using only one kind of caustics, which is either the one generated by the light rays reflected from the front surface of the specimen, or the transmitted one. Xue and Song proposed a multi-point measurement method for determining the static stress intensity factors, $K_I$ and $K_{II}$, and this is a method having relatively high precision. In this paper, we also developed a multi-point measurement method for determining the dynamic stress intensity factors, $K_{I}^d$ and $K_{II}^d$, and with this method we can investigate the dynamic propagating process of a mixed-mode crack. It is also believed that our method can be applied to opaque materials as well as transparent materials.

II. FORMATION OF CAUSTICS

The caustic technique is an optical method in experimental stress analysis. In this

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method, the singular region in the stress field is transformed into a bright curve according to a purely geometrical optical theory, and this bright curve can be received on the reference screen located some distance from the specimen. Consider a notched planar specimen of a transparent material, as shown in Fig. 1(a), and take its mid-plane to be the $x_1$, $x_2$ plane with $x_3 = 0$. A parallel light beam impinges normally to the specimen and through it. Under a load, the lateral deformation leads to a thickness reduction of the specimen. Moreover, the index of refraction is changed too. The incident light beam is divided into two components which are linearly polarized in the directions of the principal stresses, $\sigma_1$ and $\sigma_2$ in the plate, and this induces a difference in their optical path length which, for optically isotropic or inert material, is given by

$$\Delta S = c_1 (\sigma_1 + \sigma_2) d$$

where $c_1$ is the stress-optical constant of the material for the transmitted light, and $d$ the thickness of the undeformed specimen. Because of this difference in the optical path length and the inhomogeneity of the deformation in the vicinity of the crack tip, the light ray passing through the point $(x_1, x_2)$ at the specimen will deviate from a parallel path, and arrive at the reference screen behind the specimen, at the point $(X_1, X_2)$. The $X_1$-$X_2$ coordinate system is identical to the $x_1$-$x_2$ system, except that the origin of the former has been translated to $X_3 = -z_0$, where $z_0$ is the distance between the specimen and the reference screen. According to geometrical optics, the relationship between points $(X_1, X_2)$ and $(x_1, x_2)$ is

$$X_i = x_i - z_0 \frac{\partial \Delta S}{\partial x_i}, \quad i = 1, 2$$

and the condition for Eq. (2) to hold is that the lateral deformation of the specimen is much smaller than the distance $z_0$.

The resulting caustic curve on the screen is a locus of the points with multiple light rays passing through. That is, for the points on the caustic curve, the mapping (2) is not invertible, so the Jacobian of the transformation must vanish, i.e.

$$J(x_1, x_2) = \frac{\partial (X_1, X_2)}{\partial (x_1, x_2)} = 0$$

This is the necessary and sufficient condition for the existence of a caustic curve. The points on the specimen which satisfy $J(x_1, x_2) = 0$ will be mapped onto a caustic curve, while those which do not obey the condition (3) will be mapped into points.