A COUPLING MODEL OF LEFT VENTRICLE AND
ARTERIAL SYSTEM

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ABSTRACT: A simple left ventricular model and a systemic arterial model of a tapered tube with eight branches are used in this paper which are combined into a mathematical coupling model of heart and arterial system to study the interaction of the heart and the arterial system.

KEY WORDS: ventricular and vascular coupling, mathematical model, pulse wave, afterload.

I. INTRODUCTION

Almost all mathematical models[1,2,5,8,9] of the propagation of pressure and flow pulses in the arterial system simulate the function of the heart as a prescribed cyclical flow or pressure at the aortic root. These models can be used to examine how arterial geometric properties, wall properties and peripheral resistance affect the propagation of pressure and flow pulses, but they cannot be used to understand how the pressure and flow in the arterial system affect the heart performance, or how the changes of the character of the heart affect the propagation of pressure and flow pulses in the arterial system.

A very simple and effective ventricular model treating the ventricle as a thin-walled sphere was given by P.D. Corey et al.[3].

Liu Zhaorong & Zhou Yongsheng[3] studied one-dimensional non-linear transient flow through a tapered and branched tube to predict the propagation of pressure and flow pulses in human arterial system, and obtained detailed results of the pulse waves propagating along the arterial conduit extending from the heart to the tibia with eight branches.

In this paper, the two models mentioned above are combined into a mathematical model to predict the performance of the left ventricle and the hemodynamics of the arterial system. This model enables us to study mathematically how the arterial diseases affect the performance of the left ventricle, and how the heart diseases affect the pulse waves propagating in arterial system.

II. BASIC EQUATIONS

A simple and useful description of the cardiac muscular performance is provided by a three-dimensional diagram relating muscular tension \( T \), length \( l \) and contractile element shortening velocity \( V_{ce} \) (Fig. 1)[4].

This three-dimensional diagram is adopted in this paper, i.e. any point on the surface (in Fig. 1) satisfies the equation

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Fig. 1 Three-dimensional surface representing the heart muscular tension, length and shortening velocity relationship.

\[
\left( \frac{V_{\alpha}}{V_m} \right)^2 + \gamma^2 \left( \frac{T_m - T}{\gamma T_m + T} \right)^2 \left[ \left( \frac{R - R_m}{R_0 - R_m} \right)^2 + \frac{T_m^2}{(T_m - T)^2} - 1 \right] = 0
\]  

within the domain: \( 0 \leq V_{\alpha} \leq V_m \), \( 0 \leq T \leq T_m \) and \( l_0 \leq l \leq l_m \). In Eq. (1), \( V_m, T_m \) and \( l_m \) are the maximum shortening velocity, muscular tension and muscular fiber length respectively; \( l_0 \) is the minimum fiber length; \( \gamma \) is a constant parameter. The left ventricle is treated as a thin-walled sphere with inner radius \( R \). Since the radius of the ventricle is proportional to the fiber length, Eq. (1) can be rewritten as

\[
\left( \frac{V_{\alpha}}{V_m} \right)^2 + \gamma^2 \left( \frac{T_m - T}{\gamma T_m + T} \right)^2 \left[ \left( \frac{R - R_m}{R_0 - R_m} \right)^2 + \frac{T_m^2}{(T_m - T)^2} - 1 \right] = 0
\]  

The conservation of mass in ventricle yields

\[
A_v \cdot v = - \frac{d}{dt} \left( \frac{4}{3} \pi R^3 \right)
\]  

where \( A_v \) and \( v \) are the cross-section of the vessel and the blood velocity at the aortic root respectively; \( t \) is time.

The Laplace formula can be used to relate the pressure in ventricle \( (p) \) and the cardiac muscular tension \( (T) \)

\[
T = \frac{pR}{2H}
\]  

where \( H \) is the thickness of the ventricular wall.

The contractile element shortening velocity is

\[
V_{\alpha} = \frac{1}{K_m} \cdot \frac{dT}{dt} - 2\pi \cdot \frac{dR}{dt}
\]  

where \( K_m \) is a constant parameter.

Eqs. (2), (3), (4) and (5) give us four equations which include five unknown variables: \( v, R, T, V_{\alpha} \) and \( p \). The model of the arterial system must be incorporated to obtain a set of solvable equations of ventricular/arterial system.

The equations governing the one-dimensional blood flow in vessel are the continuity equation

\[
\frac{\partial (S v)}{\partial z} + \frac{\partial S}{\partial t} + \psi = 0
\]  

and the momentum equation