THE NEAR TIP FIELDS AND TEMPERATURE DISTRIBUTION AROUND A CRACK IN A BODY OF HARDENING MATERIAL CONTAINING SMALL DAMAGE*

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ABSTRACT: In this paper, the following conclusions are reached: The influence of damage on the stress and strain fields can be neglected in an asymptotic sense for the solutions of damage field in a plastic solid containing small damage. The formulation of the problem is simplified with an uncoupled approach. Based on experimental results of plastic damage, most of the damage in the material are considered as small damage with the critical damage variable $\omega_c \ll 1$. Using this approach, closed form expressions of the near tip damage fields for mode III, mode I and the temperature distribution induced by plastic dissipation in a hardening material containing damage are deduced. We point out that the temperature distribution in the process zone is strongly dependent on the damage of materials even for the small damage case. The results of the predicted value of the temperature rise near the tip region ignoring the damage effect is appreciably higher than the observed data. The main reason of this discrepancy is the presence of damage dissipation and the fact that its influence on the calculation of plastic dissipation have not been appropriately taken account of. The calculation is improved by taking into account the damage effect on the temperature rise, then the $T_{\text{max}}$ value is in better accord with the experimental value.

KEY WORDS: small damage, uncoupled procedure, near tip field, temperature distribution, damage effect.

I. INTRODUCTION

In a plastic solid described as a continuum medium, in the process of deformation until failure, the plastic damage develops through a process of nucleation, evolution and fracture characterized by the critical damage state. In recent years, the governing equations of plastic damage have been established in the framework of continuum damage mechanics\cite{1-3}. In general, the coupling effect between the stress and strain fields and the damage field must be considered, making the calculation of the damage field complicated.

There are two approaches in the calculations of damage mechanics in a continuum: first is the uncoupled procedure in which classical constitutive relation of the undamaged medium is used to predict damage and failure. As a first step, using the classical plastic theory, the stress and strain fields for plastic medium are deduced. The damage field can be obtained by substituting the stress and strain field of the undamaged medium into the damage evolution law. In this approach, the influence of the development process of damage on the stress and strain state is neglected by assumption and the calculation is simplified but error is introduced. The second way is to use the fully coupled procedure, where the interaction between the deformation and damage is considered. The constitutive relation takes into account the effect of damage

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on the deformation of solids. Then this procedure of prediction is reduced to one step in which the stress-strain and damage fields are obtained simultaneously. But up to now, only a few analytical solutions have been deduced owing to the complication of the constitutive equation. As to the numerical simulation for this kinds of coupled problems, every configuration requires a separate finite element analysis which makes computation difficult. In the calculation there also exist some theoretical and practical difficulties induced by the damage effect in the finite element method.

The main purpose of the present paper is to demonstrate a new uncoupled procedure of the analysis of plastic damage under small damage condition. The experimental results obtained by Lemaitre and Chaboche\textsuperscript{[2,3]} show that the critical damage variable $\omega_c$ is always less than 0.25 for most of the plastic metals and alloys, so the assumption of $\omega \ll 1$ is adopted throughout from the beginning of damage ($\omega = 0$) until the critical state ($\omega = \omega_c$). Therefore, damage evolution for the kind of the materials described above, or generally speaking, at the initial stage of the development of damage, the damage variable $\omega$ can be regarded as a small parameter and the problem may be investigated by asymptotic expansion. That is the meaning of the small damage concept in this paper.

After the concept about the effective stress is introduced, the conclusions that the strain field and effective stress field are not influenced by damage are demonstrated for the small damage condition by means of the asymptotic expansion method. A new-uncoupled approach is reasonable under small damage condition as a zeroth approximation of the problem in an iterative procedure. Since for most of the metals and alloys, the critical value corresponding to the plastic damage is very small, the new uncoupled approach is very useful.

Based on the considerations described above, an analogy between the governing equations of the damaged continuum medium and the undamaged solid for power law hardening materials is established. Using these relations, the fully coupled calculation of plastic damage field near the tip region for mode III crack and rigid flat inclusion and mode I plane strain case can be reduced to an uncoupled approach in the sense of asymptotic iterative procedure. The transient temperature distribution near the tip region induced by plastic dissipation with the damage effect considered is also investigated. We point out that the temperature distribution in the process zone is strongly dependent on the damage of materials even for the small damage case. The predicted value of temperature rise near the tip region ignoring the damage effect is appreciably higher than the observed data. The main reason of this discrepancy is the presence of damage dissipation and the fact that its influence on the calculation of the plastic dissipation had not been appropriately considered. The calculation is improved by taking into account the effect of damage on the temperature rise, the $T_{\text{max}}$ value is qualitatively in better accord with the observed data.

\section*{II. THE GOVERNING EQUATION OF THE PLANAR PROBLEM WITH PLASTIC DAMAGE CONSIDERED AND THE ANALOGOUS RELATIONS FOR SMALL DAMAGE}

The evolution law of plastic damage has been investigated in detail by Lemaitre and Chaboche\textsuperscript{[3]}. The evolution equation\textsuperscript{[3]} can be shown to be linear with respect to strain observed in experiments under radial loading condition.