PARAMETRIC EQUATIONS OF NONHOLONOMIC NONCONSERVATIVE SYSTEMS IN THE EVENT SPACE AND THE METHOD OF THEIR INTEGRATION*

Mei Fengxiang
(Beijing Institute of Technology)

ABSTRACT: In this paper, the parametric equations with multipliers of nonholonomic nonconservative systems in the event space are established, their properties are studied, and their explicit formulation is obtained. And then the field method for integrating these equations is given. Finally, an example illustrating the application of the integration method is given.

KEY WORDS: event space, nonholonomic nonconservative system, parametric equation, integration method.

I. INTRODUCTION

It is of importance not only in geometry, but also in mechanics to study dynamics of nonholonomic systems in the event space. In Refs. [1, 2], various parametric equations in the event space for nonholonomic potential systems are obtained. In Refs. [3, 4, 5], by means of the cyclic integral and the energy integral, the order of the parametric equations is reduced.

In this paper, the parametric equations with multipliers in the event space for nonholonomic nonconservative systems are presented. Then, some important properties of the parametric equations are studied and their explicit formulation is obtained. Finally, the field method is extended to nonholonomic nonconservative systems in the event space, and the Appell’s example in the event space is given. The main results of this paper are Eqs. (8), (9), (11), (19), (22), (23) and (30).

II. PARAMETRIC EQUATIONS IN THE EVENT SPACE FOR NONHOLONOMIC SYSTEMS

2.1 Parametric Equations

Let the position of a mechanical system be determined by n generalized coordinates \( q_s \) (s = 1, ..., n), and let its motion be subjected to g nonholonomic ideal constraints of Chetaev’s type

\[
f_\beta(q_s, \dot{q}_s, t) = 0 \quad (\beta = 1, \ldots, g; s = 1, \ldots, n)
\]

The equations of motion of the system in the configuration space can be written in the Routh’s form

\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_s} - \frac{\partial L}{\partial q_s} = Q''_s + \sum_{\beta=1}^{g} \lambda_\beta \frac{\partial f_\beta}{\partial q_s} \quad (s = 1, \ldots, n)
\]

where \( L = T - V \) is Lagrangian of the system, \( Q''_s \) are nonpotential generalized forces, and \( \lambda_\beta \) are multipliers.

Now we construct an event space \( \mathcal{R}^{n+1} \) (extended configuration space). The coordinates of
space points are \( q_s (s = 1, \ldots, n) \) \( t \), which may be regarded as totally independent variables. Let these variables be \( x_{\alpha} (\alpha = 1, \ldots, n+1) \)

\[
x_s = q_s \quad (s = 1, \ldots, n) \quad x_{n+1} = t
\]

(3)

All the variables may be given as functions of some parameter \( \tau \), which can be chosen as an arbitrary function

\[
\tau (t) \in C' \quad \frac{d\tau}{dt} > 0
\]

(4)

Let \( x_{\alpha} = x_{\alpha} (\tau) \) be some curves of class \( C^2 \) such that \( \frac{dx_{\alpha}}{d\tau} \equiv x_{\alpha}' \) are not all zero at the same time. In \( \mathcal{S}^{n+1} \), the equations of constraints are

\[
F_{\beta} (x_{\alpha}, x_{\alpha}') = 0 \quad (\beta = 1, \ldots, g)
\]

(5)

where

\[
F_{\beta} (x_{\alpha}, x_{\alpha}') = f_{\beta} \left( x_1, \ldots, x_{n+1}, \frac{x_1'}{x_{n+1}'}, \ldots, \frac{x_n'}{x_{n+1}'} \right)
\]

(6)

For a given Lagrangian \( L(q_s, t, \dot{q}_s) \), the Lagrangian of parametric form in the space \( \mathcal{S}^{n+1} \) is defined by the equality \([1,3]\)

\[
\Lambda (x_{\alpha}, x_{\alpha}') = x_{n+1}' L \left( x_1, \ldots, x_{n+1}, \frac{x_1'}{x_{n+1}'}, \ldots, \frac{x_n'}{x_{n+1}'} \right)
\]

(7)

For given nonpotential generalized forces \( Q_{st} (q_k, t, \dot{q}_k) \), the generalized forces \( P_{\alpha} \) in \( \mathcal{S}^{n+1} \) are defined by the equalities

\[
P_{\alpha} (x_{\alpha}, x_{\alpha}') = x_{n+1}' Q_{st} \left( x_1, \ldots, x_{n+1}, \frac{x_1'}{x_{n+1}'}, \ldots, \frac{x_n'}{x_{n+1}'} \right) \quad (s = 1, \ldots, n)
\]

\[
P_{n+1} (x_{\alpha}, x_{\alpha}') = - \sum_{s=1}^{n} Q_{st} x_s'
\]

(8)

The parametric equations in the event space for nonholonomic nonconservative systems can be written in the following form

\[
\frac{d}{d\tau} \left( \frac{\partial \Lambda}{\partial x'} \right) - \frac{\partial \Lambda}{\partial x} = P_{\alpha} + \sum_{\beta=1}^{g} \lambda_\beta \frac{\partial F_{\beta}}{\partial x'} \quad (\alpha = 1, \ldots, n+1)
\]

(9)

Particularly, for the case of potential forces, one has \( Q_{st} = 0 \) and \( P_{\alpha} = 0 \). Hence Eqs. (9) become

\[
\frac{d}{d\tau} \left( \frac{\partial \Lambda}{\partial x'} \right) - \frac{\partial \Lambda}{\partial x} = \sum_{\beta=1}^{g} \lambda_\beta \frac{\partial F_{\beta}}{\partial x'} \quad (\alpha = 1, \ldots, n+1)
\]

(10)

Eq. (10) are given in Refs. [1,3].

2.2 Properties of the Parametric Equations

Property 1 The Eqs. (9) are not independent, which have to satisfy the following relation

\[
\sum_{s=1}^{n+1} x_s' \left( \frac{d}{d\tau} \frac{\partial \Lambda}{\partial x'} - \frac{\partial \Lambda}{\partial x} - P_{\alpha} - \sum_{\beta=1}^{g} \lambda_\beta \frac{\partial F_{\beta}}{\partial x'} \right) = 0
\]

(11)