An Empirical Approach to the Analysis of Forest Stratification

II. Quasi-1/2 Power Law of Tree Height in Stratified Forest Communities

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Natural forest communities consist of different overlapping elementary subpopulations. Using the results of forest stratification in the preceding study, the properties of mean tree height for subpopulation in a stratified forest stand were examined. Mean tree height decreased as tree density per subpopulation increased. This relation was described by a simple mathematical model consisting of a power equation of tree density and two coefficients. The first coefficient or exponent of tree density was close to $-1/2$ in its expectation, while the other coefficient depended on life forms, especially in tropical forests. For tropical deciduous forests which suffered from seasonal forest fires, the latter coefficient was smaller than those for tropical evergreen and Japanese forests. This difference of the coefficient was not clear between tropical evergreen and Japanese forests and between deciduous and evergreen forests in Japan. In conclusion, the proposed model is similar to the 1/2 power law of tree height in man-made forests with simple architecture, and is designated the quasi-1/2 power law of tree height.

Key words: Community — Forest stratification — Quasi-1/2 power law — Subpopulation — Tree height — Tree density.

In the preceding study (Yamakura, 1987), a graphical method was empirically proposed for stratifying the individuals of a forest stand into subpopulations, using a tree height inventory of the stand. The method depended on an empirical distribution function and was successfully used for analyzing forest stratification in different forest stands in various forest formations. The conclusions on stratification which resulted from the proposed method corresponded well to the conclusions on stratification from the profile diagrams (Davis and Richards, 1933). Using the results of forest stratification in the preceding study, this paper describes briefly a new property of mean tree height in the stratified subpopulations.

Materials and Methods

In the preceding study (Yamakura, 1987), tree height data were extracted from the
studies made in natural forest stands of different forest formations in Japan, Southeast Asia, Africa, and South America. The same data were used in this study, too. All the forests were stabilized and fully matured. Further details of the data are abbreviated here (cf. Appendix I in the preceding study).

To stratify individual trees into subpopulations in a stand, the symmetric type difference diagram (Yamakura, 1987) was used. This diagram depends on the following recurrence formula of individual tree height $x$ in a stand; $x(N+n) = x(N-n) - a_n x(N) + b_n$, where $N$ is the rank of any individual in an ordered ranking of $x$, $x(1) > x(2) > \cdots > x(N) > \cdots$; $n$ is the finite difference of $N$; $x(N+n), x(N-n)$, and $x(N)$ are $x$ values labeled by the ranks, $N + n$, $N - n$, $N$, respectively; and $a_n$ and $b_n$ are coefficients specific to $n$ and the stand. On the $x(N)$ vs. $[x(N-n) + x(N+n)]$ diagram or symmetric type difference diagram, the relation between both the variables could be approximated by a few segmental linear lines, each of which was considered to correspond to a subpopulation of the total tree population in the forest stand. After stratifying individuals into subpopulations by using this diagram, the mean tree height $\bar{x}_i (m)$ and tree density $\rho_i (ha^{-1})$ in the subpopulation were calculated. In the expression of $\bar{x}_i$ and $\rho_i$, the subscript $i$ represents an identification number of a subpopulation. Here, we numbered subpopulations from $i=1$ for the lowest subpopulation to $i=I$ for the highest subpopulation. The results of forest stratification and calculated values of $\bar{x}_i$ and $\rho_i$ in this study were the same as the results given in Appendix 2 in the preceding study, since the data and numerical method for forest stratification were the same as in the preceding study. Hence, their details are also abbreviated here.

Results

Dependence of mean tree height on tree density in subpopulations in a stratified forest stand

The variable $\bar{x}_i$ increased as $i$ increased, while the variable $\rho_i$ decreased as $i$ increased in all the sample forest stands in this study. To describe this relation of $\bar{x}_i$ to $\rho_i$ in a stratified forest stand, the following power equation of $\rho_i$ was tentatively assumed,

$$\bar{x}_i = K \rho_i^h,$$

(1)

where $K$ and $h$ are the coefficients specific to the stratified forest stand. The coefficients, $K$ and $h$, are positive because $\bar{x}_i$ decreased as $\rho_i$ increased, and were estimated by the least squares method using the linear relation between common logarithms of $\bar{x}_i$ and $\rho_i$.

Fig. 1 represents the frequency distribution of the estimates of the dimensionless coefficient, $h$, from the 38 sample forest stands consisting of two or three subpopulations. Of the 50 sample forest stands, 12, 10, and 28 stands were made up of one, two, and three subpopulations, respectively. As is clear from Fig. 1, most of the estimates of $h$ are in a range from 0.2 to 0.8, and their mean and variance were 0.5191 and