MULTI-SCALE FE COMPUTATION FOR THE STRUCTURES OF COMPOSITE MATERIALS WITH SMALL PERIODIC CONFIGURATION UNDER CONDITION OF COUPLED THERMOELASTICITY*

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ABSTRACT: In this paper, the multi-scale computational method for a structure of composite materials with a small periodic configuration under the coupled thermoelasticity condition is presented. The two-scale asymptotic (TSA) expression of the displacement and the increment of temperature for composite materials with a small periodic configuration under the condition of thermoelasticity are briefly shown at first, then the multi-scale finite element algorithms based on TSA are discussed. Finally the numerical results evaluated by the multi-scale computational method are shown. It demonstrates that the basic configuration and the increment of temperature strongly influence the local strains and local stresses inside a basic cell.

KEY WORDS: two-scale method, thermoelasticity, periodic structure

1 INTRODUCTION

In the structural engineering and the development of new products, the structural problems of composite materials with small periodicity under the coupled thermoelasticity condition are often encountered. The mechanical behaviors of the structures not only depend on the macroscopic conditions, such as the geometry of the structure, the increment of temperature, the loading and constraints, but also on the basic configurations of the composite materials and the local thermal distribution in each basic cell. Therefore the detailed configuration of the composite materials and local heat behavior should be considered together as the mechanical performance of structures.

Many material, engineering experts and mathematicians have studied the problems of composite materials with small periodic configuration and illustrate typical ideas, algorithms and models[1~7]. For structures of composite materials with small periodicity under the condition of coupled thermoelasticity we have obtained the two-scale asymptotic expressions of the displacement and increment of temperature in Ref.[8]. In this paper, we discuss the multi-scale computational method for these problems. The two-scale approximate expressions of the displacement and increment of temperature for a structure are given in section 2. Then the formulations of TSA FE computation, the procedure of FE algorithms, some numerical results and conclusions are shown in sections 3, 4, 5, 6, respectively.

2 TWO-SCALE APPROXIMATE EXPRESSION OF DISPLACEMENT AND INCREMENT OF TEMPERATURE

For the coupled thermoelasticity problem

\[
\frac{\partial}{\partial x_i} \left( k_{ij} \left( x/\xi \right) \frac{\partial \theta^e}{\partial x_j} \right) = h(x) \quad \text{in } \Omega
\]

\[
\theta^e = T^0(x) \quad \text{on } \partial \Omega
\]
the two-scale asymptotic expressions of the displacement and the increment of temperature have been given in Ref.[8]. In a practical computation for the problems arisen from material sciences and engineering, one would be interested to obtain an approximate numerical solution of the first \( L \) items.

**Theorem 2.1** The structure problems (2.1), (2.2) have the following approximate solutions of the first \( L \) items in the two-scale expression

\[
\begin{align*}
\theta_\varepsilon^{(L)}(x) &= \theta^0(x) + \frac{L}{\varepsilon} \sum_{l=1}^{L} \varepsilon^l \sum_{(a) = l} \cdot \\
H_\varepsilon(x) &= H_\varepsilon^0(x) + \frac{L}{\varepsilon^2} \sum_{l=1}^{L} \varepsilon^l \sum_{(a) = l} \cdot \\
U_\varepsilon^{(L)}(x) &= U_\varepsilon^0(x) + \frac{L}{\varepsilon^2} \sum_{l=1}^{L} \varepsilon^l \sum_{(a) = l} \cdot \\
N_{\alpha}(x) &= N_{\alpha_1\alpha_2\cdots \alpha_l}(x) \\
M_{\alpha}(x) &= M_{\alpha_1\alpha_2\cdots \alpha_l}(x) \\
H_{\alpha}(x) &= H_{\alpha_1\alpha_2\cdots \alpha_l}(x)
\end{align*}
\]  

where:

1. \( \alpha = (\alpha_1, \ldots, \alpha_l) \), \( (\alpha) = |\alpha_1 + \cdots + \alpha_l| \), \( \alpha_j = 1, \ldots, n \), \( j = 1, \ldots, l \), and \( \alpha = (\alpha_1, \ldots, \alpha_l) \);

\[
N_{\alpha}(x) = N_{\alpha_1\alpha_2\cdots \alpha_l}(x)
\]

\[
(N_{\alpha_1\alpha_2\cdots \alpha_l 1}(x) \cdots N_{\alpha_1\alpha_2\cdots \alpha_l n}(x))
\]

\[
M_{\alpha}(x) = \{M_{\alpha_1\alpha_2\cdots \alpha_l 1}(x) \cdots M_{\alpha_1\alpha_2\cdots \alpha_l n}(x)\}^T
\]

\[
H_{\alpha}(x) = H_{\alpha_1\alpha_2\cdots \alpha_l}(x)
\]

For \( l = 1 \)

\[
\frac{\partial}{\partial \xi_i} \left( k_{ij}(\xi) \frac{\partial H_{\alpha_1}(\xi)}{\partial \xi_j} \right) = -\frac{\partial}{\partial \xi_i} (k_{i\alpha_1}(\xi)) \quad \xi \in Q
\]

\[
H_{\alpha_1}(\xi) = 0 \quad \xi \in \partial Q
\]

\[
\frac{\partial}{\partial \xi_j} \left[ a_{ijjk} \varepsilon_{hk}(M_0(\xi)) \right] = \frac{\partial}{\partial \xi_j} (a_{ijjk} b_{hk}) \quad \xi \in Q
\]

\[
M_0(\xi) = 0 \quad \xi \in \partial Q
\]

\[
\frac{\partial}{\partial \xi_j} \left[ a_{ijjk} \varepsilon_{hk}(N_{\alpha_1 m}(\xi)) \right] = -\frac{\partial}{\partial \xi_j} a_{ijj\alpha_1 m}(\xi) \quad \xi \in Q
\]

\[
N_{\alpha_1 m}(\xi) = 0 \quad \xi \in \partial Q
\]

For \( l = 2 \)

\[
\frac{\partial}{\partial \xi_i} \left( k_{ij}(\xi) \frac{\partial H_{\alpha_1\alpha_2}(\xi)}{\partial \xi_j} \right) = -\frac{\partial}{\partial \xi_i} (k_{i\alpha_1\alpha_2}(\xi)) - k_{\alpha_1\alpha_2} \frac{\partial H_{\alpha_2}(\xi)}{\partial \xi_j} - k_{\alpha_1\alpha_2} \frac{\partial H_{\alpha_2}(\xi)}{\partial \xi_j} \quad \xi \in Q
\]

\[
H_{\alpha_1\alpha_2}(\xi) = 0 \quad \xi \in \partial Q
\]

\[
\frac{\partial}{\partial \xi_j} \left( a_{ijjk} \varepsilon_{hk}(M_{\alpha_1}(\xi)) \right) = a_{ijjk} b_{hk} + \frac{\partial}{\partial \xi_j} (a_{ijjk} b_{hk} H_{\alpha_1}(\xi)) - k_{\alpha_1\alpha_2} \frac{\partial H_{\alpha_2}(\xi)}{\partial \xi_j} \quad \xi \in Q
\]

\[
M_{\alpha_1}(\xi) = 0 \quad \xi \in \partial Q
\]

\[
\frac{\partial}{\partial \xi_j} \left( a_{ijjk} \varepsilon_{hk}(N_{\alpha_1 m}(\xi)) \right) = -a_{ijjk} \varepsilon_{hk}(N_{\alpha_1 m}(\xi)) - a_{ijjk} \varepsilon_{hk}(N_{\alpha_1 m}(\xi)) - \frac{\partial}{\partial \xi_j} (a_{ijj\alpha_2}(\xi) N_{\alpha_1 m}(\xi)) + \delta_{ij} \varepsilon_{lm} \quad \xi \in Q
\]

\[
N_{\alpha_1 m}(\xi) = 0 \quad \xi \in \partial Q
\]  

where \( \{a_{ijjk}\}, \{\hat{b}_{\alpha_1}\} \) and \( \{\hat{k}_{ij}\} \) are called the homogenized coefficients corresponding to Eqs.(2.1) and (2.2), and can be evaluated by the following formulas from \( N_{\alpha_1 m}(\xi), M_0(\xi) \) and \( H_{\alpha}(\xi), \alpha_1, m = 1, \ldots, n \),

\[
\hat{a}_{ijhk} = \int_Q [a_{ijhk} + a_{ijlm} \varepsilon_{lm}(N_{hk})] \, d\xi \quad (2.10)
\]