Weakening (postlimiting deformation) undergone by elements of massive rock or contacts between blocks and strata plays an important part in the behavior of the rock round a working. This must be taken into account in determining the supporting capacity of pillars, the pressure on the supports, and the danger of shock bumps [1]. However, except for the simplest cases of a uniform state of stress, axial or spherical symmetry, or a thin layer, these problems cannot be solved in analytical form, and require numerical methods such as the method of finite elements (MFE), boundary integral equations (BIE), etc. Most of these methods involve discretization of the problem, i.e., the region (in the MFE) or its boundaries (in BIE) are represented in the form of a set of discrete elements. The peculiar features of the problems of weakening, consisting in the possibility of loss of stability in connection with physical but not geometrical nonlinearity, are reflected in the feature that with critical combinations of parameters the matrices representing the linear operators cease to be positive definite.

There is another important feature: Each of the discrete elements after reaching the limiting load can undergo either active irreversible deformation or load relaxation. As shown in [2], to choose between these possibilities we must invoke the method of quadratic programming (MQP).

In problems of mining geomechanics, applications of the MQP to discretized problems are only just beginning. So far we know of only one article [3] in which a combination of the MFE and MQP is used for the case of an ideally plastic medium and a piecewise-linear load surface. There are no applications to weakened rocks or to cases in which the decay diagrams reflect not the properties of volumes of the medium but rather the contact conditions. Some of these applications are described below; for the MFE we turn attention to the possibility of specific loss of stability not infrequently caused by the choice of unduly small finite elements.

1. The solution in steps with the aid of the MFE and MQP is effected as follows. Let the transition from the initial state of equilibrium to the final state be divided into several steps of increment of the external loads and displacements. These increments can be regarded as proportional to some monotonically increasing parameter t, e.g., the physical time. Then referring the increments of all the quantities to the increment of this parameter, we get the velocities (which we shall denote by a dot above the appropriate symbol). The increments of the quantities themselves in the next step are obtained by multiplying the velocities by the increment of the parameter t in this step.

In terms of velocities, in the next step we must find 
\[ \dot{\sigma}_{ij} = \dot{\sigma}_{ij}(t), \]
and the boundary conditions
\[ \dot{\sigma}_{nt} = \dot{\sigma}_{nt}(t) \text{ on } S_n, \quad \dot{\sigma}_{tt} = \dot{\sigma}_{tt}(t) \text{ on } S_t, \]
where \( \gamma_i \) are the rates of change of the bulk forces (if the latter are constant, \( \gamma_i = 0 \)); and \( \dot{\sigma}_{nt}, \dot{\sigma}_{tt} \) are the given rates of change of the loads and displacements at the boundary.

The third group of relations in (1) represents the relationships between the rates of stressing and deformation, and in general takes account of the irreversible deformations and the possibility of weakening and load relief. Without loss of generality, in this relation we can pick out the term \( \dot{\sigma}_{ij}^{(d)} \) calculated from the equa-
tions of the linear theory of elasticity. Then the term \( \varepsilon^P_{ij} \), which makes \( \varepsilon_{ij}^P \) up to \( \varepsilon_{ij} \), has the sense of the rate of plastic deformation and \( \varepsilon_{ij}^P = \varepsilon_{ij}^0 + \varepsilon_{ij}^P \), where \( \varepsilon_{ij}^0 = S^{-1} \epsilon^P_{ik} b^k \), \( S \) is the tensor of moduli of elastic yield.

If the velocities \( \varepsilon^P_{ij} \) are known in advance at each point in the body, then problem (1), (2) corresponds to the problem of the theory of elasticity with given rates of distortion \( \varepsilon_{ij}^P \). The theory of such problems has been fully developed, and they are no more difficult to solve than ordinary problems in the theory of elasticity: it is easy to show that the rates of distortion are equivalent to the mass forces \( \gamma_i \) determined from the well-known formulas [4]. This immediately becomes clear in numerical solution with the aid of the MFE: On the right-hand sides of the equations, representing the equilibrium of the elements, we find additional rates of change of the nodal forces \( \Gamma_i \), which are completely analogous to the ordinary rates of change of the nodal forces \( \Gamma_i \) arising from the bulk loads \( \gamma_i \).

For known values of \( F \) on the boundary, (3) is a system of \( 2k \) equations for \( 2k \) unknown nodal velocities. If the velocities are given at some nodes on the boundary, but not the forces, we make an obvious change of variables. For definiteness, we consider the case of given \( \dot{F} \).

Equation (3), representing a finite-dimensional approximation to the solution for a continuous medium, clearly shows that the solution \( \dot{U} \) can be written in the form of a sum \( \dot{U} = \dot{U}_e + \dot{U}_s \), in which the first term corresponds to the problem in the absence of distortion rates \( \varepsilon^P_{ij} \), \( K\dot{U}_e = \dot{F} - \vec{\Gamma}_1 \), and the second to the influence of distortion for uniform external conditions at the boundary (\( F = 0 \)) and within the body (\( \vec{\Gamma} = 0 \)) so that \( K\dot{U}_s = 0 \).

The solution to system (3) gives a global vector of rates of nodal displacement \( \dot{U} \); using this we can easily calculate the rates of deformation and the rates of stress in the elements. Their aggregate for all \( m \) elements represents the global vectors \( \dot{\varepsilon} \) and \( \dot{\sigma} \), given by \( \dot{\varepsilon} = \begin{bmatrix} \dot{\varepsilon}_{1x} & \ldots & \dot{\varepsilon}_{1y} \\ \vdots & \ddots & \vdots \\ \dot{\varepsilon}_{m x} & \ldots & \dot{\varepsilon}_{my} \end{bmatrix} \), \( \dot{\sigma} = \begin{bmatrix} \sigma_{1x} \sigma_{1y} \ldots \sigma_{1m} \\ \vdots \vdots \vdots \\ \sigma_{mx} \sigma_{my} \ldots \sigma_{mm} \end{bmatrix} \).

To exactly the same solution in the absence of distortion (\( \varepsilon^P_{ij} = 0 \)) we compare the global vectors \( \dot{\varepsilon}_e \) and \( \dot{\varepsilon}_s \), and to the solution \( \dot{U}_s \) in distortions in the absence of external actions we compare the global intrinsic fields \( \varepsilon^S, \delta^S \). The vector of the intrinsic stresses \( \delta^S \) is represented in the form of a matrix product \( Z^P \dot{\varepsilon} \), since \( \dot{U}_s \) is the matrix product \( K^{-1} \vec{\Gamma}_1 \), and the vector \( \vec{\Gamma}_1 \) is in turn linearly related to \( \dot{\varepsilon}^P \). Finally

\[
\dot{\varepsilon} = \dot{\varepsilon}' + \dot{\varepsilon}'', \quad \dot{\sigma} = \dot{\sigma}' + \dot{\sigma}'',
\]

where

\[
\dot{\sigma}' = S \dot{\varepsilon}', \quad \dot{\sigma}'' = Z \dot{\varepsilon}'', \quad \dot{\varepsilon}' = S^{-1} \dot{\sigma}' + \dot{\varepsilon}'',
\]

where \( S = \text{diag} S_r \) and \( S_r \) is the matrix of elastic moduli of the \( r \)-th element.

In deriving (4) and (5), the global vector of distortion rates \( \dot{\varepsilon}^P \) is regarded as given in advance. In this case the stress rates \( \dot{\sigma}^P, \dot{\sigma}^d \) and the deformation rates \( \dot{\varepsilon}^P, \dot{\varepsilon}^d \) at all the elements are found as a result of solution of routine problems in the theory of elasticity with the aid of MFE programs taking account of distortion. However, the rates of plastic deformation and consequently the rates of distortion are not given in advance. In each individual element \( r \) they are related by expressions which can be written in fairly general form [5] as follows:

\[
\dot{\varepsilon}^P_r = V_r \lambda_r, \quad \dot{\phi}_r = H_r \dot{\phi}_r - N_r \dot{\sigma}_r, \quad \lambda_r \geq 0, \quad \dot{\phi}_r \leq 0, \quad \dot{\sigma}_r \dot{\lambda}_r = 0,
\]

where \( H_r, V_r, N_r \) are known matrices determined from the deformation diagrams, and in general \( \lambda_r \) and \( \dot{\phi}_r \) are matrices to be determined. The physical meanings of all these quantities and of Eq. (6) were described in detail in [5]. In particular cases, \( H_r, \dot{\phi}_r, \phi_r \) can be numbers and \( V_r \) and \( N_r \) can be vectors. For an associated law of flow in the element \( V_r = N_r \).

Relation (6) is easily described for an aggregate of elements by introducing the global matrices \( H = \text{diag} H_r, N = \text{diag} N_r, V = \text{diag} V_r, \dot{\phi} = [\dot{\phi}^1_r \ldots \dot{\phi}^m_r], \lambda = [\lambda^1_r \ldots \lambda^m_r] \). Then the union of the defining relations (6) for all the elements is written in the form

\[
\dot{\varepsilon} = V \dot{\lambda}, \quad -\dot{\phi} = H \dot{\lambda} - N^T \dot{\sigma}, \quad \lambda \geq 0, \quad \dot{\phi} \leq 0, \quad \dot{\sigma} \dot{\lambda} = 0.
\]

From (4), (5), and (6) there follows a system which must be solved for \( \dot{\lambda} \) and \( \dot{\phi} \).