STRENGTH OF FISSURED ROCK AND CONCRETE
UNDER A TRIAXIAL UNIFORM LOAD

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We examine the fracture mechanism and strength of prismatic short specimens of rock or concrete subjected to brief monotonic proportional loading by a normal load in three directions (Fig. 1a).

Physical Model and General Concepts of the Fracture Mechanism. If, following the usual procedure, we take as our basis the classification of materials according to their behavior during uniform elongation of short specimens, both rocks and concrete must be classed as pseudoplastic (tensile) materials [1], for which the statistical nature of the resistance, fracture by a mechanism of gradual accumulation of damage, and sensitivity to externally introduced stress concentrators are characteristic. The principal causes of pseudoplasticity (fragmentation) are the discontinuity and nonuniformity of materials. Its principal condition is the presence of a system of defects sufficiently numerous to ensure the phenomenon of self-retardation. It is precisely the presence of such a system which reduces the tensile strength of a material or rock, but on the other hand they lose their tendency to brittle fracture.

Defects convert rocks and concretes to a system of tightly packed "grains" of random size, linked together by "bonds." Both the grains and bonds consist of the same material — the material of the "initial medium" [2], about which very little is known as yet, because each time the crucial effect on the results of tests on the specimens is exerted by the system of defects.

The principal investigation procedure is assignment of the properties of the initial medium, prediction by means of a model of the properties of the material on the scale of a sufficiently large number of grains, comparison of these properties with the experimental data, and correction of the assigned properties. To simplify the geometric form of the model of a pseudoplastic (tensile) material, we assumed the following:

The forces are transmitted from grain to grain via the bonds. Since, however, in practice transfer of compressive forces is effected by direct contact, the compressed bonds must be regarded as absolutely rigid and strong;

the bonds between the grains are located in three orthogonal directions coinciding with the directions of loading — we will call these the "principal" directions (Fig. 1b).

Previously we examined the behavior of pseudoplastic (tensile) material in the case of uniaxial elongation [2] and uniaxial compression [3] with monitoring of the final displacements of the specimen. In this article, going over to complex loadings we analyze a simpler case of a load uniformly distributed over the faces of a specimen.

According to the adopted geometric model, the force in each chain of grains is constant and is transmitted up to the appearance of damage only over the shortest distance. On the average, the load on a chain is \( \sigma_{1n} s \), where \( \sigma_{1n} \) is the nominal stresses applied in the i-th principal direction, \( s \) is the random value of the area of the mean cross section of grain, \( s \) is its expected value. The mean stresses in the grains under compression \( \sigma_1 = \sigma_{1n} s / s \), the mean stresses in the bonds and elongations \( \sigma_{1n} s / s_{bo} \), where \( s_{bo} \) is the random value of mean cross-sectional area of the bonds.

We take \( \sigma_{1n} > \sigma_{2n} > \sigma_{3n} \) and assume positive compressive stresses. With proportional loading \( \sigma_{3n} / \sigma_{1n} = \omega_2, \sigma_{2n} / \sigma_{1n} = \omega_3 \), where \( \omega_2 \) and \( \omega_3 \) are constants. For actual stresses, \( \sigma_2 / \sigma_1 = \omega_2 \) and \( \sigma_3 / \sigma_1 = \omega_3 \), respectively.

When \( \sigma_{1n} = 0 \), the bond energy of this model is independent, and from the strength conditions \( \sigma_{3n} < R_t \) in the presence of uniaxial tension there follow the strength conditions \( |\sigma_{2n}| < R_t, |\sigma_{3n}| < R_t \) in the presence of...
Fig. 1. Short prismatic specimen: a) applied forces; b) geometric model; 1) grains; 2) bonds.

Fig. 2. Bond energy between grains: a) adjoining grain chains; b) mutual shear of half-grains; 1) intact grains; 2) fractured grains.

of biaxial tension; and $|\sigma_1| < R_t$, $|\sigma_2| < R_t$, and $|\sigma_3| < R_t$ in the presence of triaxial tension. Experiments do not give significant deviations from these conditions [4-6]. Thus the task of the investigation reduces to elucidation of the fracture mechanism and determination of the strength conditions in five other possible cases, for which $\sigma_1 > 0$:

1. $\sigma_1 > 0$; $\sigma_2 > 0$; $\sigma_3 = 0$;
2. $\sigma_1 > 0$; $\sigma_2 = 0$; $\sigma_3 < 0$;
3. $\sigma_1 = 0$; $\sigma_2 > 0$; $\sigma_3 < 0$;
4. $\sigma_1 > 0$; $\sigma_2 < 0$; $\sigma_3 > 0$;
5. $\sigma_1 < 0$; $\sigma_2 < 0$; $\sigma_3 < 0$.

Owing to the fact that the grain cross sections are random, even in the presence of small nominal stresses there is the possibility of overloading of individual grains and their separation into two parts as a result of fracture due to shear in the weakest direction. The possibility of such fracture before the onset of fluidity in the grains is typical of nonuniform media, like rocks and concrete [7-9, 3].

Separation of the grains into parts leads to a change in the state of stress. Following Gvozdev [10], we represent it as the superposition on the principal stress field of a secondary field of self-balanced stresses, leading to fracture of the specimen by separation into parts over the cross section normal to the third principal direction.

We will imagine that the specimen is slit by a plane in the $3$ direction. Figure 2a shows two adjoining grain chains in such a cross section. The fractured grains (2) form dipoles, which induce compressive forces in the $3$ direction. From the beginning these forces may be balanced by the external load of this direction. However, as the number of fractured grains increases this load will be insufficient and tension will appear in the bonds (the minus sign in Fig. 2a). The supporting power will be attained when the overall forces in the elongated bonds attain the limiting value.

Since graphs of the work of pseudoplastic (tensile) materials have a descending sector, for any other stressed states the "stress-strain" curves will have such a sector.

Secondary Stresses. The degree of nonuniformity of an actual stress state depends crucially on the pattern of the statistical distribution of the mean grain cross sections. Stereographic analysis of a system of tightly packed grains, on the assumption that the distribution of the number of grains obeys Poisson's law, leads to an equation for the distribution density $p$ [11]

$$p(s) = \frac{1}{s} \cdot \frac{(\mu s)^{k} e^{-\mu s}}{\Gamma(k)},$$

where $\mu = 1/\langle s \rangle$, and $k$ is a parameter. Below we take $k=3$.

We do not have direct data on the grain strength. In the simplest case of uniaxial compression we postulated that shear takes place along the direction of $\tau_{\text{max}}$, when these stresses attain the limiting value independent of the hydrostatic pressure [3]. In the general case of complex loading there is a greater probability of the appearance of shear along a direction close to one of the octahedral areas.

Experiments reveal the influence of the intermediate principal stress on the strength of rocks and concretes. This necessitates a certain broadening of the strength conditions of the grains, assuming [12]:

$$\tau_{\text{oct}} = \tau_0 + k_0 \sigma_{\text{oct}},$$

where $\tau_0$ and $k_0$ are constants characterizing the mechanical properties of the initial medium.