ROCK BREAKING

REFLECTION OF A NONSTEADY WAVE FROM
THE INTERFACE OF TWO NONIDEAL-ELASTIC MEDIA

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It is known that when a nonsteady wave is incident on the interface of two media, in some cases we get infinite stresses (in the linear theory) at a point moving with the loading front (the intersection of the wave front with the interface). Corresponding results have been obtained for the cases of water and water [1], water and an elastic medium [2, 3], elastic medium and elastic medium (stationary case) [4], and liquid and elastic layer [3].

In addition, stress singularities can arise at a point moving along the interface (including the free surface of an elastic half-space) at the velocity of the Rayleigh wave; however, we will not be considering this case in the present article.

This article is devoted to an investigation of the influence of the properties of the medium (viscosity and other types of nonideality) on the formation and development of singularities at a point moving with the loading front.

§1. Formulation of the Problem. Consider an elastic space divided by the plane $z = 0$ into two parts, in which the properties of the medium are in general different. To investigate this problem, let us consider the following conditions [3]: starting at time $t = 0$, from some straight line $x = 0$ on the interface the normal load (the action of the direct wave) is propagated in both directions along the $x$ axis (where $x$ is the coordinate in the plane of the interface). We can take the problem to be symmetrical in this way because the solution in the neighborhood of the singularity in the left-hand part ($x < 0$) will depend only on the action of the wave on this part of the plane, because disturbances from the right-hand part ($x > 0$) of the plane will not, when they reach the left-hand part, contain singularities; these will have been "smoothed out." Further, since the formation of a singularity is influenced only by the value of the load in the immediate neighborhood of the front, we shall consider the load, where it exists ($|x| \leq c_0t$), to be constant ($c_0$ being the velocity of the load).

If the load is generated by a wave, it must be calculated on the assumption that the interface does not move [5]. When reflection occurs, a tangential load also arises. Here we consider only the action of the normal load. The complete solution is made up of this together with the solution of the problem of the action of the tangential load alone.

Since we are finding the solution with the aid of the Laplace integral transform, in the following argument, in the images of the solution, the constant moduli of the bulk compression and shear can be replaced by the images of the corresponding operators (multiplied by the parameter $p$ of the transformation) with respect to the media in which the relation (local with respect to the coordinates) between the stresses and deformations takes the form of a convolution [8].

Let

$$\sigma_{ij} = \delta_{ij} \left[ \frac{d \theta(t)}{d \tau} \left( k(t - \tau) - \frac{2}{3} \mu(t - \tau) \right) d \tau + 2 \int_0^t \frac{d x_{ij}(\tau)}{d \tau} d \tau \right], \tag{1}$$

where
\[ \varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \]
\[ \Theta = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}, \]

\( k \) is the bulk compression coefficient, \( \mu \) is the shear modulus, and \( \delta_{ij} \) is the Kronecker delta (\( i, j = 1, 2, 3 \)).

Applying the Laplace transform [7] to Eq. (1), we get
\[ \sigma_{ij}^t = pk^t \Theta^t \delta_{ij} + 2p\mu^t \varepsilon_{ij}^t - \frac{2}{3} \mu^t \Theta^t \delta_{ij}. \]

Here and below, \( \mathcal{L} \) is the Laplace transform with respect to \( t \) (with parameter \( p \)), \( \mathcal{F} \) is the Fourier transform with respect to \( x \) (with parameter \( q \)). If \( k \) and \( \mu \) are constants, then their Laplace transforms are \( k/p \) and \( \mu/p \), i.e., from Eq. (1) we get Hooke’s law,
\[ \sigma_{ij}^0 = k \delta_{ij} + 2\mu \varepsilon_{ij}, \]

From Eqs. (3) and (4) it follows that to obtain a solution in images for Eq. (1) we must replace the constant moduli \( k \) and \( \mu \) in the solution for the elastic problem by \( pk^L \) and \( p\mu^L \), respectively.

On this basis let us formulate the conditions of the problem. On the interface plane there acts a load \( \sigma_0 \),
\[ \sigma_0 = g_0 \delta_0 (c_0 - |x|), \]

where \( \delta_0 \) is the Heaviside unit function, \( g_0 \) is a dimensional constant. The boundary conditions on the interface plane (\( z = 0 \)) involve continuity of the displacements and tangential stresses and a fixed discontinuity in the normal stress (by the above-mentioned load),
\[ \sigma_0^+ = \sigma_0^-, \quad \sigma_0^+ = \sigma_0^- + \sigma_0^0. \]

Here and below, \( \mathcal{F} \) applies to the medium at \( z > 0 \), and \( \mathcal{F} \) to the medium at \( z < 0 \). At infinity there are no disturbances. The initial conditions are zero.

§2. Solution of Equations of the Theory of Elasticity. Applying the Laplace integral transform and the Fourier transform successively to the equations of the dynamics of a volume element of a continuous medium, without taking account of mass forces, except for inertia, and to the equations of Hooke’s law relating stresses to deformations, and using the boundary and initial conditions, we get a solution in image stresses,
\[ \sigma_{xx}^{LF} \big|_{z=0} = L_1/L, \quad \sigma_{zz}^{LF} \big|_{z=0} = L_2/L, \]
\[ L_1 = -2ig_0 \rho p^+ q^2 \left( n_1^+ + n_1^- \right), \]
\[ L_2 = 2g_0 \rho^+ \left[ 2q^2 (c_1^{+3} - c_2^{+3}) (\rho^{-} T_1 - \rho^+ T_2) - \rho^2 (\rho^{-} T_3 - \rho^+ T_3) \right], \]
\[ L = c_0 n_0^2 \left[ 2q^2 R (\rho^{-} T_1 - \rho^+ T_2) + \rho^2 (\rho^{-} T_3 + T_4) - \rho^+ T_3 - \rho^{-} T_3 \right], \]
\[ R = \rho^+ (c_1^{+3} - c_2^{+3}) - \rho^- (c_1^{-3} - c_2^{-3}), \]
\[ T_1 = q^2 - n_1^+ n_2^+, \quad T_2 = q^2 - n_1^- n_2^-, \quad T_3 = q^2 + n_1^- n_2^+, \quad T_4 = q^2 + n_1^+ n_2^-, \]
\[ n_i^+ = \sqrt{q^2 + b_i^+ \rho^2} \left( n_i^+ > 0 \right. \text{ when } p > 0, q = 0), \]
\[ b_i^+ = 1/c_i^+ \left( i = 0, 1, 2 \right), \]
\[ c_i^+ = \sqrt{q^2 + 4\mu i / \rho^2}, \quad c_i^- = \sqrt{\mu i / \rho^2}, \]

where \( \rho^\pm \) is the density of the medium.