Service life analysis from field data on age distributions

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ABSTRACT
In survey investigations of building stocks for example, data on age distributions of construction details are often collected. What interests the material scientist, though, is not the age distribution but rather the service life distribution. Presented herein will be a renewal process model describing the relationship between such an age distribution found and the corresponding service life distribution being sought. The model comprises a phenomenological parameter to accommodate correlations between objects and the service life's determining factors. However, it is shown that, because of numerical ambiguity, the correlation parameter cannot be uniquely determined from an age distribution. Therefore, some additional information and data processing are required before the full advantage of this model can be realized. This somewhat difficult part has yet to be completed. Nevertheless, in its present form, the model does point out possible ranges of service life distribution parameters in terms of the correlation strength, thus providing uncertainties in the distribution parameters, which up until now have normally been overlooked when estimating the service lives of building materials. For wide service life distributions, it is shown that the possible parameter ranges will also be rather wide.

1. INTRODUCTION
The results presented herein are closely related to and have evolved from ideas emanating from the work performed in RILEM TC 140-TSL on Prediction of Service Life of building materials and components.

Let us consider a certain part of an item (e.g. a building or any other construction), which will be repaired or replaced many times during the item's entire service life. Such parts, belonging to a stock of items, as defined by proper means, are characterised by a specific, but still unknown, service life distribution. Furthermore, suppose that at a certain time, a sampling is taken of this stock. The parts of the items sampled constitute the set of objects to be analysed. Data on the age (the time since the previous maintenance, or, if this has not occurred, the time since the item was erected) of each object is collected. Obviously, there is a correspondence between the age distribution of the objects analysed and the service life distribution being sought. In the following, the

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analytical relationship between these distributions will be given; its validity however will depend on the fulfillment of the following conditions:
- steady-state conditions, i.e. for a long period, compared to the mean service life of the objects, the number of items within the stock as well as the distributions of the factors affecting service life (loads, material qualities, maintenance methods and failure criteria) have been unaltered;
- discarding of objects where maintenance has not been carried out, in spite of the performance of these objects surpassing any acceptable failure criterion; and
- if it is anticipated that the service life distributions differ between fresh and repaired parts, the objects must be separated into two independent groups: one consisting of replaced parts or parts belonging to new items, and another consisting of repaired parts.

2. MODELLING

2.1 Randomly-distributed homogeneous service life

Randomly-distributed homogeneous (RDH) service life means that the longitudinal service life distribution equals the transversal distribution, i.e. the distribution of consecutive service lives (maintenance intervals) for a specific object is the same as the service life distribution for all objects at an instant in time. This is an idealised picture, one that is hardly found in the real world, since factors affecting the service life (loads, material qualities, maintenance methods and failure criteria) are usually object-correlated (e.g. dependent on the local environment or on preferences of the owners/maintenance companies).

At time 0, let \( a(t) \) be the probability density function (PDF) of the age \( T_A \) of the objects, where the age is defined as the time since the previous maintenance. Specifically, \( a(0) \) is the rate of objects maintained at time 0. At a later time, say \( t_1 \), this density has been reduced to:

\[
a(t) = a(0) \int_t^{t_1} f(t') dt'
\]  

(2)

where \( a(0) = 1/N \) can be considered as a normalising factor given by:

\[
N = \int_0^{\infty} \int_0^{\infty} f(t') dt' = \int_0^{\infty} \int_0^{t} f(t') dt' = \int_0^{\infty} t f(t') dt'  
\]  

(3)

\( N \) can be recognised as the mean of \( T_L \). Thus, the maintenance rate \( a(0) \) is the inverse of the mean service life, which of course is to be expected. Further details can be found in [1], for example.

2.2 Constant longitudinal - randomly-distributed transversal service life

Constant longitudinal - randomly-distributed transversal (CL - RDT) service life is the other extreme where the service life is a constant in time for a specific object, while there is a distribution in service life within the set of objects. This means that for a specific object, the maintenance interval will always be the same, which may be even more unrealistic than the RDH model. Nevertheless, in order to set a limit, it is valuable to find the expression for the age PDF in terms of the transversal service life PDF.

An object with a service life \( t' \) will certainly receive maintenance each interval of \( t' \) time units. Accordingly, at any time, the rate of maintenance for objects with service lives between \( t' \) and \( t' + \Delta t \) will be:

\[
t'^{-1} f(t') \Delta t
\]  

(4)

where \( f(t) \) now denotes the transversal service life PDF of the objects. Let \( b(t) \) be the age PDF at time 0, for which, as before, \( b(0) \) equals the maintenance rate at time 0. Therefore:

\[
b(0) = \int_0^{\infty} t' f(t') dt'  
\]  

(5)

At \( t_1 \) time units later, the density of objects on which maintenance was performed at time 0 has been reduced, since objects with shorter service lives than \( t_1 \) have not survived. Obviously, this fraction is:

\[
\int_0^{\infty} t' f(t') dt' = \int_0^{\infty} t' f(t') dt' - \int_t^{t_1} t' f(t') dt' = \int_t^{t_1} t' f(t') dt'  
\]  

(6)

Thus the difference between equations (5) and (6) yields the remaining density at \( t_1 \), which as before, because of the steady state, equals the density of objects at time 0 but on which maintenance was performed \( t_1 \) time units ago. Then:

\[
b(t) = \int_0^{\infty} t' f(t') dt' - \int_t^{t_1} t' f(t') dt' = \int_t^{t_1} t' f(t') dt'
\]  

(7)

The absence of a normalisation factor is verified by:

\[
\int_0^{\infty} \int_0^{\infty} t f(t') dt' dt = \int_0^{\infty} f(t') dt' = 1
\]  

(8)

(equation (8) can be evaluated by using equation (3) and multiplying the integrand by \( 1/t' \).)