It is known that a real rock mass has a laminar structure, as a rule. Experiments show that in the general case all its parameters depend on the depth. The need to take account of this dependence is resolved separately in each problem. If the scale of the region is sufficient for the appearance of inhomogeneity over the depth, it must be taken into account. The general case in which the mass is a half-space consisting of individual layers is considered here. Each layer has its own elastic parameters: Young's modulus $E$, Poisson's ratio $\sigma$, density $\rho$. Suppose that all the layers are parallel to the bottom surface $x_3 = 0$ (as usual, $x_1$, $x_2$, $x_3$ are Cartesian coordinates) and within each layer the elastic parameters depend explicitly on the depth, i.e., the functions $E(x_3)$, $\sigma(x_3)$, $\rho(x_3)$ are piecewise-smooth, with discontinuities at the layer boundaries. The known direct methods of estimating the elastic parameters and stress state of the mass [1] become inapplicable or expensive at large depths. In such cases, geophysical methods associated with the solution of inverse problems of dynamic elasticity, for example, may be used. In such problems, a dynamic influence (explosion, shock, vibrational load) is specified at the surface, with measurement of the displacement at the free surface of the mass over some time interval. Since the speed of the longitudinal and transverse waves is finite, and the source is instantaneous and concentrated at a point, the perturbation covers only some finite region. At points of the surface $x_3 = 0$, the displacement vector $v(x, t)$ is measured at $0 < t \leq T$. The basic aim of the present work is to determine the parameters of the mass from experimental data on the displacement vector.

The inverse dynamic problem associated with determining the multidimensional characteristics of weak inhomogeneous inclusions in a laminar elastic medium was considered in [2]. The problem of determining the one-dimensional characteristics $E(x_3)$, $\sigma(x_3)$, $\rho(x_3)$ within a single layer was investigated in [3-5].

Consider the problem of determining $E(x_3)$, $\sigma(x_3)$, and the density $\rho(x_3)$ under the assumption that the elastic medium is multilayer. To this end, the Lamb problem associated with the construction of the displacement field $v(x, t) = (v_1(x, t), v_2(x, t), v_3(x, t))$ arising from the action of an instantaneous directed force concentrated at the coordinate origin at the boundary $x_3 = 0$ is considered. The functions $v_i(x, t)$, $i = 1, 2, 3$, satisfy the system of differential equations of dynamic elasticity theory

\[
\rho \frac{\partial^2 v_i}{\partial t^2} = \sum_{j=1}^{3} \frac{\partial}{\partial x_j} \left\{ \mu \left( \frac{\partial v_j}{\partial x_j} + \frac{\partial v_i}{\partial x_i} \right) + \delta_{ij} \lambda \text{div} v \right\}
\]

with initial conditions

\[
v|_{t=0} = 0,
\]

\[
\left[ \mu \left( \frac{\partial v_i}{\partial x_i} + \frac{\partial v_i}{\partial x_3} \right) + \delta_{ij} \lambda \cdot \text{div} v \right]|_{x_3=0} = -\frac{1}{2} \delta(x_i) \delta(x_j) \delta(t)
\]

and coupling conditions at the interfaces of the medium

\[
|v_i| |_{x_3=l+i} = 0, \quad \left[ \mu \left( \frac{\partial v_i}{\partial x_i} + \frac{\partial v_i}{\partial x_3} \right) + \lambda \delta_{ii} \text{div} v \right]|_{x_3=l+i-0} = 0.
\]

The functions $\lambda(x_3)$, $\mu(x_3)$ are Lamé elastic parameters.
In Eqs. (1)-(4), the following notation is used: $\delta(\cdot)$ is a delta function; $\delta_{ij}$ is the Kronecker delta. The boundary condition in Eq. (3) models a directed force concentrated at the point $(0, 0, 0)$; the condition in Eq. (4) indicates that rigid-contact conditions hold at the layer boundaries $x_3 = \xi_i$, $i = 1, n$.

Consider the inverse one-dimensional Lamb problem: to determine the twice continuously differentiable functions $\lambda(x_3)$, $\mu(x_3)$, $\rho(x_3)$ with finite discontinuities at the layer boundaries $x_3 = \xi_i$, $i = 1, n$, from the following information

$$\int_0^L \int_{x_1}^{x_2} v_2(x_1, x_2, 0, t) e^{i(v_1 x_1 + v_2 x_2)} \, dx_1 \, dx_2 \, |_{v_1 = 0, v_2 = \kappa} = h_j(t),$$

$$\int_0^L \int_{x_1}^{x_2} v_3(x_1, x_2, 0, t) e^{i(v_1 x_1 + v_2 x_2)} \, dx_1 \, dx_2 \, |_{v_1 = 0, v_2 = \kappa} = h_3(t).$$

Here the functions $h_k(t)$, $k = 1, 2, 3$, are known functions when $0 < t \leq T$; $\kappa_1 = 0$; $\kappa_2$ is a specified nonzero number. Fourier transformation with respect to the variables $x_1, x_2$ with the transformation parameters $v_1, v_2$ is applied to Eqs. (1)-(4). Suppose that $V = F_{x_1, x_2}[v]$ ($v_1, v_2, x_3, t$), where $v$ is a vector function which is a solution of Eqs. (1)-(4).

Let $v_2 = 0$, $v_1 = \kappa_j$, $j = 1, 2$, where $\kappa = 0$, $\kappa_2 = \nu \neq 0$; then the system of equations for the vector function $V$ breaks down into two subsystems. One consists of a differential equation for $V_2(x_j, 0, x_3, t)$

$$\rho \frac{\partial^2 V_2}{\partial t^2} = \frac{\partial}{\partial x_3} \left( \mu \frac{\partial V_2}{\partial x_3} - \kappa_2^2 V_2 \right)$$

with the initial conditions

$$V_2|_{t < 0} = 0, \quad \mu(0) \frac{\partial V_2}{\partial x_3} \bigg|_{x_3 = +0} = -\frac{1}{2} \delta(t)$$

and conditions at the layer boundaries

$$\left[ V_2 \right]_{x_3 = \xi_i+0}^{x_3 = \xi_i-0} = 0, \quad \left[ \mu \frac{\partial V_2}{\partial x_3} \right]_{x_3 = \xi_i+0}^{x_3 = \xi_i-0} = 0.$$ (7)

When $v_1 = v_2 = 0$, the second system breaks down into two independent parts. The third equation in terms of the function $V_3 = F_{x_1, x_2}[v_3](v_1, v_2, x_3, t) |_{v_1 = v_2 = 0}$ takes the form

$$\rho \frac{\partial^2 V_3}{\partial t^2} = \frac{\partial}{\partial x_3} \left( (\lambda + 2\mu) \frac{\partial V_3}{\partial x_3} \right)$$

with the initial conditions

$$V_3|_{t < 0} = 0, \quad (\lambda(0) + 2\mu) \frac{\partial V_3}{\partial x_3} \bigg|_{x_3 = +0} = -\frac{1}{2} \delta(t)$$

and conditions at the layer boundaries

$$\left[ V_3 \right]_{x_3 = \xi_i+0}^{x_3 = \xi_i-0} = 0, \quad \left[ (\lambda + 2\mu) \frac{\partial V_3}{\partial x_3} \right]_{x_3 = \xi_i+0}^{x_3 = \xi_i-0} = 0.$$ (10)

In the given notation

$$V_2|_{v_1 = 0, v_2 = \kappa_j, x_3 = +0} = h_j(t), \quad j = 1, 2,$$ (11)