OPTIMAL ORE SORTING AUTOMATA

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A machine separating a primary ore mass into classes will be called an ore-sorting automaton. For the purpose of this study, the design features and the physical operation principles of the machine are immaterial; we are only concerned with its behavior function — the capability of dividing an ore mass into portions according to a certain characteristic, thus implementing a given separation function.

The ore mass to be sorted will be viewed as a space \((X, \mathcal{A}, \mu)\), where \(X = \{x\}\) is a discrete set of partition elements (particles, lumps, or portions) constituting the ore mass; \(\mathcal{A}\) is the set of all possible subsets consisting of the elements \(x \in X\), where the set \(\mathcal{A}\) is a \(\sigma\)-algebra: It contains the set \(X\) and the empty set and is closed with respect to the union and complement operations performed in a countable number; \(\mu\) is the Lesbegue measure, which for definitiveness can be taken as the weight of the ore enclosed in any \(A \in \mathcal{A}\). In this study we will assume that the weight of ore enclosed in \(X\) is finite, and, moreover, that \(\mu(X) = 1\). With this assumption the space \((X, \mathcal{A}, \mu)\) can be identical to the probability space \((X, \mathcal{A}, P)\), where \(P\) is the probability measure on \(\sigma\)-algebra of the subsets from \(X\), \(P\{A\} = \mu\{x: x \in A \subseteq X\}\).

Suppose that on elements \(x \in X\) the function \(\xi(x)\) has been defined, which is a physical or technicoeconomic variable used as the criterion of sorting. Conventionally, such variables are referred to as the division characteristic; accordingly, \(\xi(x)\) is a division characteristic. The behavior of an elementary or sorting automaton can be described by the characteristic function

\[
\theta(x) = \begin{cases} 1, & \text{if } \xi(x) - \chi > 0, \\ 0, & \text{if } \xi(x) - \chi \leq 0, \end{cases}
\]

where \(\chi\) is a threshold value functioning as the controlled parameter.
An ore-sorting automaton of this kind for any fixed threshold $\chi$ divides the ore $X$ into two nonoverlapping classes: $X_\chi = \{x: \theta(x) = 1\}$ and $X\setminus X_\chi = \{x: \theta(x) = 0\}$. The separation function on the set $X$ is defined by the equality $\theta(x) - \chi = 0$.

An exhaustive description of the ore with respect to its separation according to the $\xi$ feature is the $\xi$ distribution function:

$$F(\chi) = F(\xi(x) < \chi) = \mu(X_\chi) = \mu(x: \xi(x) < \chi).$$

The distribution function can be defined constructively. Suppose the threshold variable $\xi$ runs successively through all the values from the interval $(\xi_*^{*}, \xi^*)$, forming an ordered sequence

$$\xi_* = \chi_1 < \chi_2 < \ldots < \chi_m = \xi^*,$$

where $\xi_* = \min \xi(x)$, $\xi^* = \max \xi(x)$ are the least and the largest values of $\xi$ on the elements $x \in X$.

Isolating for each value of $\chi$ from that series with the aid of (1) the corresponding subset $X_\chi \subset X$, we obtain a set of embedded sets ordered by inclusion:

$$\emptyset \subset X_{\chi_1} \subset X_{\chi_2} \subset \ldots \subset X_{\chi_m}$$

and an ordered sequence of ore weights contained in each of the sets of that sequence:

$$0 \leq \mu(X_{\chi_1}) \leq \mu(X_{\chi_2}) \leq \ldots \leq \mu(X_{\chi_m}).$$

If now we put into correspondence to each threshold from sequence (2) an ore weight from sequence (3) $\chi \to \mu(X_\chi)$, we will obtain a mapping $F: \{\chi\} \to \{\mu(X_\chi)\}$ which represents the distribution function $F(\chi) = \mu(X_\chi)$.

The amount of ore contained in each set $X_\chi$ is linked with the characteristic function of that set by the relation

$$\mu(X_\chi) = \sum_{\mu(\chi) < \chi} \mu(x) = \sum_{x \in X} \theta(x) \mu(x).$$

The distribution function is monotonic and nondecreasing; it is defined only by the properties of the ore and is an exhaustive physical characteristic of ore separability according to the $\xi$ feature for the given partition. On the other hand, it can be viewed as a static characteristic of the "ore-separation automaton" system in the action-output control channel, where the control factor is the threshold value of the ore-sorting machine. Importantly, the automaton is ideal and defines without error the value of the division feature $\xi$ at each $x \in X$ and operates without malfunction.

The distribution function $F(\chi) = \mu(X_\chi)$, by virtue of the law of conservation of matter and the condition $\mu(X) = 1$, predetermines the existence of a different function $\mu(X\setminus X_\chi)$ as a measure of the difference of the sets $X\setminus X_\chi$, which expresses the quantitative yield of the second product. Obviously, $\mu(X\setminus X_\chi) = 1 - \mu(X_\chi)$.

The distribution of $\xi$ with respect to the set of elements $X$ can also be characterized by a distribution density: the function $\Psi(x)$, connected on each $\mu$-measurable $A \in \Theta$ with the distribution function by the integral relation

$$F(A) = \int_A \Psi(x) \, d\mu.$$

An ore-sorting automaton is usually a component of a more complex separation system — the separation stage (Fig. 1) — which produces concentrates of metal products. The efficiency of a separation stage from a certain point of view can always be estimated by a criterion. In a general form, the efficiency criterion can be represented by a sign-variable measure $\phi$ defined on each ore subset $A \in \Theta$. Strictly, the assumption that the functions representing the efficiency criterion and used as the efficiency functions of control are of a variable sign has to be proved; the proof is not given here. To confirm that it is sign-variable we will merely mention that if a function $\phi$ describing the goal were not a sign-variable measure on $\sigma$-algebra of the subsets from $X$, the problem of optimum control of an ore-sorting automaton according to that criterion would not exist. The control would be reduced to the basic choice between directing all or no ore to the next separating stage.