USE OF THE METHOD OF FINITE ELEMENTS TO INVESTIGATE 
THE STATE OF STRESS OF A LINING WITH AN IRREGULAR 
CROSS SECTION

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In the construction and repair of underground workings, much use has recently been made of progressive gunite (sprayed concrete) linings. As a rule, gunite is used in strong, stable rocks with higher moduli of elasticity than the lining and outside the zone of influence of other workings, so that the support plays the part of an insulating coat. The extensive introduction of gunite under other conditions is prevented by lack of knowledge of the interactions in the lining-rock system.

Existing analytical solutions [1-3] refer to the stress distribution round an unsupported working with an uneven surface or round a working with an absolutely rigid lining [3]. Experiments on models of optically sensitive materials, in which the influence of unevenness of the periphery of the cross section of the lining on the tangential stresses was studied, have been performed at the All-Union Scientific-Research Mine Surveying Institute [4]. The external shape of the lining was assumed to follow a hypocycloid. The cases $E_I/E_r>1$ (2.86) and $E_I/E_r<1$ (0.32) were considered, together with two types of internal periphery (circular and with constant lining thickness).

In this article we give the results of an investigation of the state of stress of a lining with an irregular cross-sectional surface in relation to the ratio between the elastic characteristics of the lining-rock system ($E_I/E_r = 1$) and to the dimensions and shape of the lining and the layer of weakened rock beyond the lining, the weak layer being modeled by an elastic layer with reduced modulus of elasticity.

The calculations were performed by the method of finite elements (MFE) in the elastic formulation by means of a program written at the B. E. Vedeneev All-Union Scientific-Research Institute of Hydrotechnics [5] for the BESM-4 computer.

The MFE, applied to problems of mining geomechanics, is a fairly effective numerical method of constructional mechanics [6]. The precision of a solution by the MFE depends on the denseness of the network at the points of maximum stress gradient. As in [4], we considered the case of an axisymmetric load applied at the boundary of the region in question after installation of the lining. The solution of this problem can be applied to the analysis of the action of the lining of mine shafts and the support of horizontal workings with a uniformly distributed load ($q$) applied at the infinite boundary of the region.

In a working supported with gunite, the surface of the lining which is in contact with the rock repeats the relief of the rock walls, while the interior surface has a broadly similar shape but with a smaller amplitude of fluctuation. According to the results of processing field measurements, the outline of the lining can be represented by epitrochoidal curves [7] with parametric equations of the form

\[
x = R (\rho \cos \theta - m \rho^n \cos n\theta);
\]

\[
y = R (\rho \sin \theta - m \rho^n \sin n\theta),
\]

where $\theta$ is the angle measured from the direction of the $x$ axis to the direction of a ray passing through the point with coordinates $(x, y)$, $R$ is the mean radius of the ring for which the area of the convexities (hollows in the rock)
Fig. 1. Scheme of division of region into triangular elements. a) Lining with weak layer; b) solid rock.

Fig. 2. Distributions of stresses in lining ($\sigma_\theta$) and contact pressures ($\sigma_r$). a) $C_1 = 1.1$; b) $C_2 = 1.05$.

I. $E_l = E_r$
II. $E_l = 5E_r$; $E_r = E_w$
III. $E_l = 10E_r$
IV. $E_l = 5E_r$; $E_r = 10E_w$.