THE DISPLACEMENT FIELD AROUND A WORKING OF CIRCULAR CROSS SECTION

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Improvements to methods of analyzing underground supports are closely linked with study of the stress-strain (stress-deformation) state of the rock around a working. Investigations based on the theory of elasticity have revealed that stress redistribution occurs in the rock as a working is being driven. The new state of stress is determined by the initial state of stress of the rock, the geometry of the working, and the mechanical characteristic of the support. In the case of a long working at a depth more than five times its radius, we can base our calculations on a weightless plane weakened by a hole, the shape of which corresponds to the cross section of the working [1], with stresses at infinity equal to the stresses in the undisturbed rock at the center of the future working.

In this article we will give a numerical calculation of the displacement field in the plastic zone in the case of the plane viscoelastic problem of tension (compression) of a plane with a circular hole.

Let us consider a plane weakened by a circular hole of unit radius, with forces at infinity equal to \( \sigma_x^\infty = p \), \( \sigma_y^\infty = q \), \( q \geq p \). For given values of \( p \) and \( q \) this viscoelastic problem has a unique solution [2]. The viscoelastic boundary is an ellipse with semiaxes

\[
c(1+\delta), \quad c(1-\delta),
\]

where

\[
c = e^{\frac{p+q}{4k} \frac{1}{2}}; \quad \delta = \frac{q-p}{2k},
\]

and \( k \) is the plasticity constant.

In the elastic zone the stresses, deformations, and displacements are known. Let us determine the deformations, and from them let us find the displacements \( D_p \) in the plastic zone. For this purpose, as shown in [3], we must determine the function \( \Phi(r, \theta) \) from problems (1)-(2):

\[
\frac{\partial^2 \Phi}{\partial r^2} - \frac{1}{r^3} \frac{\partial^2 \Phi}{\partial \theta^2} + \frac{3}{r} \frac{\partial \Phi}{\partial r} = 0; \tag{1}
\]

\[
\left. \frac{\partial \Phi}{\partial r} \right|_{r=r_0(\theta)} = -\frac{1}{\lambda + \mu} \frac{r^2 + r'^2}{r^2 - r'^2} \left[ \frac{\partial \psi}{\partial r} \right] - \frac{1}{2\mu} \left( \sigma_\theta - \sigma_r \right)_e - \frac{2rr'}{r^2 - r'^2} \frac{1}{\mu} \left[ \frac{\partial \psi}{\partial r} \right]; \tag{2}
\]

where \( \lambda \) and \( \mu \) are the Lamé constants and \( \sigma = 1/2 \left( \sigma_r + \sigma_\theta \right) \); \( [\psi] \) is the discontinuity in function \( \psi \) at the transition through the viscoelastic boundary. Suffix \( e \) denotes the derivative in the elastic region.

The deformations \( \varepsilon_\theta, \varepsilon_r \) are related as follows to the unknown function \( \Phi(r, \theta) \):

\[
\varepsilon_\theta = \frac{4k \ln r + 2k}{4(\lambda + \mu)} + \frac{k}{2\mu} + \frac{1}{2} \Phi(r, \theta); \quad \varepsilon_r = \frac{4k \ln r + 2k}{4(\lambda + \mu)} - \frac{k}{2\mu} - \frac{1}{2} \Phi(r, \theta); \quad \varepsilon_r = 0.
\]
Let us construct an approximate solution to problem (1)-(2) in the form

\[ \Phi(r, \theta) = A_0 + \frac{B_0}{r^2} + \sum_{n=1}^{N} \left[ A_n \sin \left( \frac{1}{4n^2} - 1 \ln r \right) + B_n \cos \left( \frac{1}{4n^2} - 1 \ln r \right) \right] \frac{\cos 2n\theta}{r}; \]

\[ \sin(n, r) = \sin \left( \sqrt{4n^2 - 1} \ln r \right); \]

\[ \cos(n, r) = \cos \left( \sqrt{4n^2 - 1} \ln r \right). \]

The unknown constants \( A_0, B_0, A_n, \) and \( B_n \) are found from the boundary conditions (2).

In view of the symmetry of the problem it is sufficient to consider the interval \( 0 \leq \theta \leq \pi/2 \), dividing it into \( (N + 1) \) parts, whence we obtain \((2N + 2)\) linear algebraic equations for the \((2N + 2)\) unknowns. In the particular case \( p = q \) we find the exact solution

\[ \Phi(r, \theta) = A_0 + \frac{B_0}{r^2}. \]

After solving the system we determine the function \( \Phi(r, \theta) \) and hence the deformations, from which by the usual method we recover the displacements.

By the above method we calculated the displacement field in the plastic zone around a working in a medium with the parameters \( k = 40 \text{ kg/cm}^2, \lambda = \mu = 72,000 \text{ kg/cm}^2 \). In Tables 1-4 we list the radial and tangential displacements \( u \) and \( v \), calculated at the periphery of a working for various stresses at infinity (in kg/cm²):