A STEEPEST-DESCENT ALGORITHM FOR DETERMINING
THE OPTIMUM PARAMETERS OF VENTILATION
NETWORKS IN POTASH MINES

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At present there are several methods for determining the optimum parameters (the cross sections of the mine workings) of a ventilation network [1-3]. However, it must be admitted that the methods in [2] and [3] are cumbersome, have elements of complex analysis, and are thus difficult to realize on a computer. The linearization method [1] is simpler, but it has the disadvantage that it involves having initially given working cross sections (usually standard). This makes it difficult to use in calculating parameters for networks in potash mines.

In this article we give a method of multistage search—a method of steepest descent and an algorithm for realizing it for a given class of functions.

As a rule, economic extremal problems of search for the most profitable variant of a ventilation network are formulated as follows.

Minimize \( \varphi(S_i) \)
with the conditions \( g_j(S_i) = h \)

\[ j = 1, \ldots, m; \ i = 1, \ldots, n. \]

where the functional (the objective function) \( \varphi(S_i) \) represents the cost of cutting, maintaining, and ventilating workings with cross sections \( S_i \). The functional is linear relative to the desired quantities \( S_i \) if it does not include costs of ventilation. Constraint (2) is a set of equations which take account of the law of distribution of air over the ventilation network and represent the sum of the pressure drops: if condition (2) is stated for closed contours, \( h = 0 \); if it is stated for open routes, \( h \) is the depression developed by the main ventilation fan; \( m \) is the number of closed contours and open routes, and \( n \) is the number of parameters to be found.

The method of steepest descent, based on determination of the gradients of a functional, gives rapid convergences if the following conditions are satisfied: the objective function (1) is nonlinear with respect to the desired parameters and is convex above; condition (2) is linear with respect to the desired parameters, which, in view of the linearity of (2), are a convex set.

A new form of the problem satisfying these conditions can be obtained if we solve it, not for the cross sections \( S_i \), but for the relative resistances of the workings, which in turn are functions of the \( S_i \):

\[ x_i = \Psi(S_i), \]

where the \( x_i \) are the resistances of workings of unit length in \( k \mu / m \).

Thus we get a new problem:

Minimize \( F(x_i) \)
with the conditions

\[ f_j(x_i) = h, \ j = 1, \ldots, m, \]
\[ x_i \geq 0, \ i = 1, \ldots, n. \]
In calculations on ventilation networks in potash mines, the target function (3) does not contain the cost of maintenance of the workings, because as a rule workings in potash mines are not maintained. In the new formulation of the problem, function (3) has the general form

\[ F(x) = \sum_{i=1}^{n} K_i x_i^{-0.4} - \sum_{i=1}^{n} T_i x_i + \sum_{i=1}^{\delta} T_i x_i, \]

where \( K \) is the cost in rubles of cutting workings with a resistance of \( \mu \) m, \( T \) is the cost in rubles of moving the air, and \( t \) is the number of additional resistances \((x_t)\) due to extra air control structures (stoppings, ventilation doors) necessary to create the required air distribution.

The first derivative of function (6) with respect to \( x \) is

\[ F_x(x) = -0.4 \sum_{i=1}^{n} K_i x_i^{-1.4} - \sum_{i=1}^{n} T_i + \sum_{i=1}^{t} T_i, \]

and the second derivative is

\[ F_{xx}(x) = 0.56 \sum_{i=1}^{n} K_i x_i^{-2.4}. \]

When \( x_i \geq 0 \), (8) is positive; consequently, by the argument in [4], function (6) is convex below, and therefore in realizing problem (3)-(5) by the method of steepest descent we have a global minimum [5].

Suppose that the point \( x_0 = \{x_0, \ldots, x_n\} \) is a permissible initial solution satisfying conditions (4); then the best approximation can be found if we move along the antigradient with some step \( \lambda \). To determine a possible suitable direction of descent to the minimum, we solve an auxiliary problem in linear programming:

Maximize \( -F_x(x^0, y) \)

with the conditions \( f_j(y) = h, y \geq 0 \),

where \( -F_x(x^0, y) \) is the scalar product of the antigradient of the function \( F(x) \) with some quantity \( y \).

A possible suitable direction of descent is defined as the difference

\[ u = y - x^0. \]

The greatest difficulty is presented by the search for a finite step length \( \lambda \). Some authors suggest that as \( \lambda \) we should take a value which will not bring the solution \( x^1 = x^0 + \lambda u \) outside the permissible range; i.e., the value of \( \lambda \) must not be too great (because the process may diverge). If \( \lambda \) is too small, we will require a large number of iterations (steps in the calculation).

In any case \( \lambda \) must be such that

\[ F(x^0 + \lambda u) \rightarrow \min \]

in the best way. For a given initial value \( x^0 \) and direction \( u \),

\[ F(x^0 + \lambda u) = F(\lambda), \]

i.e., the unknown is the step length \( \lambda \). Function (13) has a minimum at the point \( \lambda \) satisfying the condition

\[ \frac{dF(\lambda)}{d\lambda} = 0. \]

If function (3) is quadratic, expression (14) will be linear with respect to \( \lambda \), and it will be easy to solve.

In our case, function (3) takes the form (6). Then the step length \( \lambda \) can be found by applying Newton's method to Eq. (14),

\[ \frac{dF(\lambda)}{d\lambda} = F_\lambda(\lambda) = 0, \]