CAVING OF A CANTILEVERED ROOF

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We will consider the problem of the coupling of upper and lower elastic half-planes between which a semi-infinite seam and a cantilever lying on it are included. We will derive the condition relating the span of the working with the length of the overhanging cantilever, its thickness, and the thickness of the worked seam. Equating the flexure of the overhanging roof with that of the cantilever and relating this flexure to the distributed load on it, we obtain enough relations to construct the curve relating the bending moment of the cantilever to its length. Account is taken of the plastic zone near the face of the seam.

Suppose that a worked seam of thickness \( h \) lies at depth \( H \) from the surface. We assume that the process of working the seam is accompanied by block caving of the cantilevered hanging roof of thickness \( h_b \) above the waste area. The caving rocks of the roof and floor smoothly join up with the caved layers covering the floor (see Fig. 1).

With this system of coordinates, the state of stress in the rock before the working is driven is

\[
\sigma_x^0 = -\gamma (H - y); \quad \sigma_y^0 = -\gamma (H - y); \quad \tau_{xy}^0 = 0. \tag{1}
\]

When the working is present, the stresses in the rock can be written as a sum,

\[
\sigma_x = \sigma_x^0 + X_x; \quad \sigma_y = \sigma_y^0 + Y_y; \quad \tau_{xy} = X_y, \tag{2}
\]

where \( X_x, Y_y, \) and \( X_y \) are the additional stresses due to the working. These additional components satisfy the system of equations

\[
\begin{align*}
-\frac{\partial X_x}{\partial x} + \frac{\partial Y_y}{\partial y} &= 0; \\
\frac{\partial X_y}{\partial x} + \frac{\partial Y_x}{\partial y} &= 0; \\
\Delta (X_x + Y_y) &= 0.
\end{align*}
\]

In the complex plane \( z = x + iy \), the expressions for the stress and displacement components can be written [1] in the form

\[
\begin{align*}
X_x + i X_y &= \Phi (z) + \Phi (\overline{z}) - (z - \overline{z}) \overline{\Phi'(z)} - \overline{\Omega(z)}; \\
Y_y - i X_y &= \Phi (z) + \Phi (\overline{z}) + (z - \overline{z}) \overline{\Phi'(z)} + \overline{\Omega(z)}; \\
2\mu \left( u + l \psi \right) &= x \varphi (z) - z \varphi'(z) + \psi (z); \\
2\mu \left( \frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x} \right) &= x \Phi (z) - \Phi (\overline{z}) - (z - \overline{z}) \overline{\Phi'(z)} - \overline{\Omega(z)},
\end{align*}
\]

where

Fig. 1

\( \Phi (z) = \psi' (z); \quad \Psi (z) = \psi' (z); \)

\[ \Omega (z) = z \Phi' (-z) \quad \Omega (\infty) = \Phi (\infty) = 0; \]

\( u \) and \( v \) are the displacement components; \( v \), measured on the real axis, determines the position of the floor and roof in the presence of the working.

We assume that there are no tangential stresses along the roof and floor surfaces before or after the working is driven, i.e.,

\[
X_y = 0 \quad (y = 0; \quad \infty < x < \infty). \tag{5}
\]

The above problem falls into two composite problems for the upper and lower half-planes. At the boundary of the upper half-plane \((y = 0)\) we have the following conditions. To the left of the point of contact of the rocks, \(x = 0\), the vertical displacements are determined by the thickness of the caved rock layer, \(\lambda h_0\), i.e.,

\[
v^+ (x, 0) = \lambda h_0 \quad (-\infty < x < 0), \tag{6}
\]

where \(\lambda\) is the coefficient of loosening of the rock. In the waste area from the point of contact \(x = 0\) to the overhanging roof \(x = x_1\), the surface is free from external loads, i.e.,

\[
\sigma_y^+ (x, 0) = 0 \quad (0 < x < x_1). \tag{7}
\]

By (1) and (2), this condition, for additional stress \(Y_y\), can be written as

\[
Y_y^+ (x, 0) = \gamma H \quad (0 < x < x_1). \tag{8}
\]

In the section of the overhanging cantilever up to the face \(x = x_2\), the normal component \(\sigma_y\) is determined by the reaction of the cantilever \(N(x)\), i.e.,

\[
\sigma_y^+ (x, 0) = -N (x) \quad (x_1 < x < x_2), \tag{9}
\]

or else for \(Y_y^+\) we have

\[
Y_y^+ (x, 0) = \gamma H - N (x) \quad (x_1 < x < x_2). \tag{10}
\]

In the remaining part of the roof \((x > x_2)\) we assume that the displacements normal to the seam are equal to some constant \(h_1\),

\[
v^+ (x, 0) = h_1 \quad (x_2 < x < \infty), \tag{11}
\]

which is determined by the condition of coupling of the upper and lower half-planes.

For the lower half-plane we have the following boundary conditions. To the left of the point of contact,

\[
v^- (x, 0) = 0 \quad (-\infty < x < 0); \tag{12}
\]

in the section of waste area from the point of contact to the face,

\[
\sigma_y^- (x, 0) = 0 \quad (0 < x < x_2) \tag{13}
\]

or, by (1) and (2),

\[
Y_y^- (x, 0) = \gamma H \quad (0 < x < x_2); \tag{14}
\]

beyond the face \((x > x_2)\)

\[
v^- (x, 0) = h_2 \quad (x_2 < x < \infty), \tag{15}
\]

where \(h_1\) and \(h_2\) are related as follows

\[
h_1 + h_2 = h + h_b. \tag{16}
\]

Conditions (5)-(13) completely determine the composite problems for the upper and lower half-planes without allowance for plasticity of the seam near the face. However, as in [2], in the course of solving the problem the plastic zone near the face will be completely defined and its influence on the state of stress of the roof will be discussed.

From (3) with condition (5) we have