GENERAL PROBLEMS OF METROLOGY AND MEASUREMENT TECHNIQUE

ACCURACY PROBLEMS AND SCALE THEORY. INTERPRETATION OF MEASUREMENT ERRORS

L. Z. Dich

The possibility is considered of applying scale theory (representative theory) to the description of measurement accuracy. It is proposed that the measurement error should be considered as an independent object which is described by constructing its own scale which may not coincide with that constructed in order to describe the object of the measurement itself. It is noted that the thesaurus used in metrology needs to be extended.

A comparison was made in [1] between the approaches used in scale theory (the representative theory of measurements) and in classical metrology for describing measurement procedures. An analysis is now made of the possibility of applying the methods of scale theory to a description of measurement error.

Measurement Error and the Formalism of Scale Theory. The correctness of a representation constructed in scale theory when formalizing a measurement procedure implies, in particular, that it is unique and that each represented state corresponds to one and only one image number. Actually, however, any measurement algorithm which is practically implemented cannot guarantee such uniqueness. Moreover, even if such a condition were observed then, for example, the nonlinearity of representation which unavoidably exists in practice would prevent one from speaking of the construction of an isomorphic or homomorphic image. The total effect of these factors leads to a situation in which a measurement procedure creates a nonunique image of reality, i.e., according to [2] a state \( a \) of a measured characteristic \( x \) is represented in sets of values of an abstraction region: \( a \Rightarrow B_a = \{b\} \), where \( \{b\} \) is some interval (subset) of numerical values. Since in [2] the property of isomorphism of representation is initially postulated, its author is unavoidably forced to ask whether the postulate corresponds to things as they really are. Here it is mentioned that indeterminacy of representation can in principle be made compatible with the formal requirement of isomorphism, but only for an extremely specific case in which one is essentially concerned with the representation of a countable set of states. The question of the representation of a continuum, which precisely relates to the overwhelming majority of measurements, remains open.

In this case the fact of the matter is that one cannot ignore the existence of measurement error as part of the proposed formalism and that in order to utilize this concept it is necessary to find logical foundations which are no less weighty than those for the measurement concept itself.

If one poses this question in the way it was done in [2], i.e., understanding the measurement process to be a process of nonunique representation, then it will be necessary to admit that a quantity \( Q \) is represented into a set of value-error pairs \( \{x, \delta x\} \) or into a set of intervals \( [x', x''] \). For example, instead of a representation \( r : Q \rightarrow R_+ \) it is necessary to construct a representation \( r : Q \rightarrow R_+^2 \). Here there are two objections. Firstly, the inclusion of an error in a numerical model describing the properties of an object is not justified since it in no way characterizes the properties of the object but characterizes only the measurement process itself. Secondly, \( n_1 \)-dimensional and \( n_2 \)-dimensional sets cannot be isomorphic only if the condition \( n_1 = n_2 \) is satisfied. Consequently, \( R_+ \) and \( R_+^2 \) cannot be isomorphic and neither can an isomorphism relationship be established between \( Q \) and \( R_+^2 \). Thus, neither the set of pairs which was mentioned nor the set of intervals can serve a satisfactory model of reality.
Taking into account that “a measurement is not an aim in itself but only a means of achieving the aim” [3] the existence on nonuniqueness of representation forces one to ask the questions:

1. Is one justified in attempting to construct a formal model in which the properties of an object are represented not in number but in numerical intervals?

2. If one is not justified in doing this, is there any sense at all in invoking scale theory in order to describe measurement errors even taking account of the advantages it gives when describing such measurements?

3. If one is justified, how can one still correctly introduce the concept of error?

If it turns out, for a positive answer to the first question, that it proves to be possible to construct the model being sought, then the rationality of the succeeding possible steps is doubtful. The fact of obtaining some number as a result of a measurement procedure very frequently proves to be only the first step in a chain of further actions which, in particular, can result in calculations or in the generation of control instructions. It is very difficult to imagine a situation in which, for given purposes, one would employ not a single result number but a whole numerical interval. The latter, at the very least, would result in a sharp increase in the volume of the calculations and would additionally cause considerable difficulties when solving poorly conditioned problems and problems in which the result of the calculations does not depend in a continuous way on the value of the measured quantity.

It makes sense to consider a representation model, such as a scale, only in interrelationship with other abstract models which utilize numbers. Any analytical dependence into which the measured value of a quantity is inserted will adequately describe reality only to the extent that it itself depends on another model, on a numerical image of the object. It is for this reason that, however rough the measurements, in the calculations one uses only their result itself as though it had been obtained as part of an ideal representation. The final accuracy of the calculations or control actions is estimated separately. Thus, it is not very productive to seek a correct description of errors by constructing nonunique representations.

In order to answer questions 2 and 3 let us turn our attention to the difficulties which arise when determining the error of a number of types of measurement by classical methods.

In the case of an automatic calculation of the result of indirect measurements, \[ z = z(x_1, x_2, ..., x_j) \] it is impossible to restore the original values of the input quantities \( x_i \) with which the measurement occurred or the calculation was made [4]. Meanwhile, one and the same value \( z \) can be obtained for different sets of values \( (x_1, x_2, ..., x_j) \) which correspond to different error values \( \delta z \). In this case it is impossible uniquely to compare any accuracy with the result \( z \) despite the fact that the errors \( \delta x_i \) in each case can be indicated uniquely. On this basis it was even proposed that one should not at all class corresponding measuring instruments as means of measurement. The diametrically opposite opinion was expressed in [5, 6] where it was proposed to consider an instrument (for example, a wattmeter) as a means of direct measurements if all the calculating operations are performed within the instrument while the operator taking down the readings is not involved in the calculations but records only the final result. Nevertheless, the problem of determining the error \( \delta z \) of the readings from such instruments does not disappear when this is done.

Another example is associated with a cited problem of monitoring the shape of a surface. It is well known [7] that when describing the inaccuracy of manufacturing a surface having some given nominal shape, one indicates the distance from points on the real surface to some reference surface which is constructed in accordance with some or other fixed rules and whose shape coincides with the nominal shape. When using this method to estimate the shape, it becomes impossible to distinguish between a distortion of the actual shape and a displacement of this surface in space without a distortion of its shape.

In optical manufacturing, when monitoring the shape of optical surfaces, test glasses are utilized as a standard surface, but it is effective to use them only when the radii of the tested and standard surfaces are practically equal. In addition, in this case also they resort to concepts of general and local errors of shape, giving a pair of interference fringe numbers \( (N, \Delta N) \). This description can be informative only in combination with the observation of the interference pattern whose shape contains additional albeit qualitative information. It is very difficult to judge the quality of manufacture of a surface by relying solely on this pair of numbers and such monitoring of the shape of a surface requires a detailed description of the rules governing its behavior. In particular, the form of the interference pattern (rings, fringes) and the number of fringes observed can be stipulated. This however is an additional veiled normalization.

A variation in monitoring the shape of a surface is that of making multiple measurements of the diameter of a cylindrical or spherical object in different directions. Thus, the assumptions are introduced that circles or spheres are unique geometric objects whose sizes are constant for any direction of measurement. However, one can contrast so-called constant thickness curves and bodies with these objects. The simplest example is that of the Reuleaux triangle and the corresponding body of