METHOD OF RECONCILING THE RESULTS
OF SUBJECTIVE MEASUREMENTS

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A new method of reconciling the results of subjective measurements to make them consistent, based on identification of local areas in the quotient space of nonuniformly consistent expert estimators, is considered. The problem of data self-correction is solved together with establishment of the properties of connectivity and transitivity of the relations between measurements. The method may be used for intelligent analysis of data and for improving the quality of decisions.

By subjective measurements we will understand the process of arriving at the values of the indicators of the properties of complex systems as a result of a specially constructed survey of a single individual or of a group of individuals.

Depending on the level of an individual's knowledge in the application domain formed by systems that are being considered, we may provisionally distinguish between ordinary subjective and expert estimators. Preference is given to expert estimators since they are close to the objective estimators. Expert estimators are used in the solution of applied problems in decision methods whenever it is not possible to conduct measurements or obtain objective estimators. If there are no experts available, ordinary subjective estimators are used. The latter estimators always contain measurement errors the magnitude of which exceeds that of the errors in the expert and in the objective estimators. Since most people are not experts in arriving at practical decisions, reducing the errors in subjective measurements down to the level of the errors that are permitted by experts or down to the level of the errors of objective measurements is of critical importance in the problem of measurement.

The mechanism of subjective measurements and the technique followed in its implementation are determined by the method of representation of knowledge about the properties of the object. Knowledge may be expressed in the form of judgments and relations. Below, we will consider subjective estimators of the properties of systems represented in the form of relations. Through a representation of estimators in the form of preference relations it becomes possible to compare not easily formalized system properties that are different in terms of physical nature, as well as alternative systems, in terms of each of their properties. The mechanism used to represent the properties of systems in the form of hierarchies together with the process of obtaining normalized coefficients of significance for all the properties of systems on each level of the hierarchy have been referred to as the method of hierarchy analysis [1]. This method makes use of paired comparison in dominance matrices that have the following form:

\[
M = \begin{bmatrix}
1 & W_{12} & W_{13} & W_{14} \\
W_{21} & 1 & W_{23} & W_{24} \\
W_{31} & W_{32} & 1 & W_{34} \\
W_{41} & W_{42} & W_{43} & 1
\end{bmatrix} = \begin{bmatrix}
a & a & a & a \\
b & 1 & c & b \\
c & c & 1 & c \\
d & d & d & 1
\end{bmatrix},
\]

where \( a, b, c, \) and \( d \) are unknown absolute estimators of the properties of the object; and \( W_{12}, W_{13}, \ldots, W_{43} \) are known subjective estimators that constitute the preference relations of properties.

Through representation of subjective estimators in the form of dominance matrices it becomes possible to establish a connection between estimators from different rows and columns of the matrix; determine the eigenvector of the matrix; and

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TABLE 1. Weights and Ranks of Objects Obtained from Results of Weighing

<table>
<thead>
<tr>
<th>Object</th>
<th>Actual relative weight</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radio receiver</td>
<td>0.104</td>
<td>4</td>
</tr>
<tr>
<td>Printer</td>
<td>0.406</td>
<td>1</td>
</tr>
<tr>
<td>Large attache case</td>
<td>0.208</td>
<td>3</td>
</tr>
<tr>
<td>Projector</td>
<td>0.281</td>
<td>2</td>
</tr>
</tbody>
</table>

Subjective estimators that are used as the initial information in a mathematical approach based on generating the eigen-vector of the matrix (1) may prove to be consistent or inconsistent.

A technique based on calculation of the numerical value of the indicator of the consistency relation of the estimators (CR) [1, 2] is used to estimate the degree of consistency of the initial data. The technique is widely used in methods of hierarchy analysis:

\[ CO = IC / RC; \]  
\[ IC = \frac{\lambda_{\text{max}} - n}{n - 1}; \]  
\[ \lambda_{\text{max}} = R_1 \sum_{i=1}^{n} W_{ij} + R_2 \sum_{i=1}^{n} W_{ij} + \ldots + R_n \sum_{i=1}^{n} W_{ij}; \]

\[ R_i = \left( \frac{\prod_{j=1}^{n} W_{ij}}{\sum_{j=1}^{n} \prod_{j=1}^{n} W_{ij}} \right)^{-1}, \]

where IC is the index of consistency of expert estimators; RC is the random consistency of expert estimators of a matrix of dimension n [2]; \( \lambda_{\text{max}} \) is the greatest eigenvalue of the inversely symmetric matrix \( M \); \( n \) is the number of columns and rows of the matrix; \( i \) is the index of the row of matrix \( M \); \( j \) is the index of the column of matrix \( M \); and \( W_{ij} \) are the expert estimators.

A value of \( CO \) less than or equal to 0.10 [1] is considered acceptable, and solutions may be obtained from the initial data. If the value of \( CO \) exceeds the admissible level (a value found for \( CO \) greater than \( CO_{\text{adm}} = 0.10 \)), the initial information unacceptably distorts the decision maker (contradiction in the information greater than some norm) [3, 4]. In this case, the adopted solutions will be characterized by considerable inaccuracy.

To prove this assertion, we present an example that illustrates how to select the lightest object from among a set of comparable objects, thus illustrating the influence of the level of consistency of subjective estimators on the quality of the ultimate solution. Suppose that, as a result of objective measurements (weighing on scales), the weight of a radio receiver, of a printer, of a large attache case, and of a projector have been obtained. It is necessary to determine their actual relative weight and rank and to find the lightest object.

The actual relative weight of an object is determined by dividing its measured weight by the sum of the weights of all the objects. The ranks of the objects are determined by comparing the absolute or relative weights. Since the initial data were obtained objectively, the estimators of the preference ratios are consistent and \( CO = 0 \). The actual relative weights and ranks of the objects are shown in Table 1. From an analysis of these data it follows that the radio receiver is the lightest object.

Let us vary the conditions of the experiment. Suppose that there are no scales available. We will use subjective expert measurements. As a result of polling an expert subject, the estimators presented in Table 2 are obtained.