We present a method for computing the value of the gravitational constant with subsequent storage of information in a compact form; the method allows for all possible variations in the measurements. The procedure permits presentation of the material in a form that is convenient for study and further research.

Measurement of the gravitational constant $G$ with torsion balances is reduced to measurement of time intervals between pulses generated by an optical system [1]. Time intervals can be measured very accurately by means of frequency meters, and they can be recorded with digital recorders. The functions of a frequency meter and digital recorder can be performed by a PC, provided two signals are applied to an input port: a stable quartz-crystal frequency and a sequence of electrical pulses with steep leading edges. Such pulses are generated by 521CA3 microcircuits, whose input receives the signal from a photodiode. The use of a computer makes it possible to obtain information immediately about time intervals in electronic form on magnetic media, which considerably simplifies both the measurement process and processing of the measurement results with special software that computes and saves results of measurements of the gravitational constant $G$.

Method for Processing Measurement Results. For each position of attracting masses, the program for computation of the gravitational constant $G$ uses five time intervals that are undistorted by the process; these variables are used to compute the period and amplitude of the oscillation and the gravitational constant. When the oscillation amplitude is small, the gravitational constant can be computed either with a simplified formula obtained by expansion of the moment of attractive forces in a series with allowance for the fifth power of $\varphi$ (\(\varphi\) is the angle of deviation of the suspension from the equilibrium position), or it can be computed from a differential equation. When the amplitude is large, we use only the second approach, which, if we have additional information about the period of oscillation of the weights in the absence of attractive forces, can be used with the Runge–Kutta method to determine the period of anharmonic oscillations. We then choose a value for $G$ that compensates for the difference in the inverse squares of the computed and experimental periods of oscillation of the balance with the positions of the attracting masses recorded at two points.

Constants Used for Computation. In computing $G$, in addition to ten time intervals, we must have information about a group of constants that are kept constant while conducting a series of measurements of $G$. Measurements are conducted over a variety of intervals that may be as long as 6 months. The use of a longer series is complicated by the possibility that corrections may be required: restoration of a high degree vacuum, elimination of errors that may have occurred during the experiment, optimization of balance parameters, replacement of attracting masses, etc.

The group of constants used in the computation include the following: $m_1$, the mass of the suspension pendulums; $m_2$ the mass of the suspension; $M$, the difference between the masses of an attractive sphere and the air it displaces; $J_2$, the moment of inertia of the working object; $L_1$, $L_2$, $L_3$, $L_4$ the distances from the axis of rotation to the center of mass of the attracting bodies when their positions are recorded at three or four different points; $L_5$, the distance from the axis of rotation to the center of mass of the pendulum of the suspension; $L_6$ the length of an arm of the suspension; $c_1$, the coefficient correcting the period when there is a change in the amplitude of the oscillation (due to the presence of gradients in the gravitational field); $c_2$, the constant allowing for the position of the photo sensors relative to the null position of the balance (in the case of an asymmetric scheme,
\(c_2 = 1\); in the case of a symmetric scheme, \(c_2 = 2\); \(c_3\) is the constant for the optical system – this constant is used to compute the amplitude of oscillations; \(n\) is the number of segments into which the arms of the suspension are divided; \(h\) is the vertical displacement of the attracting masses from axis of the suspension; \(T\) is the period of oscillation of the balance without attracting masses.

**Computation of \(G\) and the Type of Information Storage.** In order to compute \(G\) and subsequently store the accumulated information, we wrote special software to process results of measurements made with the instrument described in [2]. This software package allows for all of the variations that may occur over the entire measurement period. Measurements were initially made with a single-cycle system using a single attracting mass. Subsequently we only used two-mass systems.

Initially, one of the measured time intervals may be consistent with the phase of \(\varphi = 0\), and in this case we realize an asymmetric measurement scheme (\(c_2 = 1\)); here the first and last of the five registered time intervals and the sum of the three remaining intervals are approximately equal to half the period of oscillations in the balance. When \(c_2 = 1\), the displacement process for the attracting masses does not initially match the longest time interval.

In a symmetric scheme (\(c_2 = 2\)), measurements may begin anywhere in the oscillation of the balance. This scheme was preferred, since for \(c_2 = 1\) there was a chance that the experiment would fail as a result of an incorrect selection of the phase of oscillation. The group of five intervals – three short \((t_1, t_3, t_5)\) and two long \((t_2, t_4)\) intervals – makes it possible to determine the average value for two oscillation periods that are separated by the interval \(t_{1i}\) (the index \(i\) indicates the position of the attracting masses). Final values for the periods \(T_i\) and the amplitude \(\varphi_0\) are computed with the formulas

\[
T_i = 0.5 (t_{1i} + t_{5i} + t_{2i} + t_{4i} + t_{3i})
\]

\[
\varphi_0 = c_3 \sqrt{\frac{c_2}{\sin \pi (t_{2i} + t_{4i}) / (c_2 T_i)}}.
\]

where \(c_3\) is the optical-system constant used to compute the amplitude of the oscillations. In the absence of noise, \(t_{5i} < t_{1i}\), and the difference \(t_{1i} - t_{5i}\) is proportional to the damping decrement.

After \(T_i\) and \(\varphi_0\) are determined, we compute the value of the gravitational constant for all possible combinations of positions for the attracting masses in a measurement cycle. A measurement cycle includes the positions for which the attracting masses are displaced in one direction (toward or away from the balance). The measurements at the extreme positions are common to two consecutive cycles. The gravitational constant is computed with two independent methods. In one method, we use the differential equation of motion

\[
d^2\varphi / dt^2 + \omega^2 \varphi + GM \left[ m_1 L_i (b_1 + b_2) \sin \varphi + m_2 (b_3 + b_4) / (b_5 + b_6) \right] / J_3,
\]

where

\[
b_1 = L_3 \left( L_5^2 + L_7^2 - 2 L_8 L_5 \cos \varphi + h^2 \right)^{-3/2};
\]

\[
b_2 = -L_3 \left( L_5^2 + L_7^2 + 2 L_8 L_5 \cos \varphi + h^2 \right)^{-3/2};
\]

\[
b_3 \equiv \left[ (L_8 + L_5 \cos \varphi)^2 + h^2 \right]^{1/2} / \left[ L_6 \left( L_7^2 + L_8^2 + 2 L_6 L_7 \cos \varphi + h^2 \right)^{1/2} \right],
\]

\[
b_4 \equiv \left[ (L_8 - L_5 \cos \varphi)^2 + h^2 \right]^{1/2} / \left[ L_6 \left( L_7^2 + L_8^2 - 2 L_6 L_7 \cos \varphi + h^2 \right)^{1/2} \right].
\]

\(\omega = 2\pi / T\) is the cyclic frequency of oscillation in the absence of attracting masses.

In the other method of calculating \(G\), we use the system of difference equations