EVALUATING THE PARAMETERS OF THE SIGNAL
OF A LASER-DOPPLER VIBROMETER

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The method of maximum similitude is used to estimate vibration characteristics on the basis of the signal obtained from a laser-Doppler vibrometer. The accuracy of the estimates is evaluated on the basis of statistical modeling, the results of which are also presented.

In measuring mechanical vibrations with a laser-Doppler anemometer (LDA), it is very important to consider the parameters of its signal. Those parameters make it possible to characterize the motion of the object being studied. The electrical signal at the output of an LDA has the form of a phase-modulated oscillation. When the reflecting surface moves in accordance with a certain law \( \phi(t) \), the signal can be described by the expression:

\[
U(t) = U_0(t) \cos(\alpha \phi(t) + \varphi_0),
\]

where \( U_0(t) \) is the accompanying amplitude modulation of the signal; \( \varphi_0 \) is the initial phase of the vibrations; \( \alpha \) has the sense of amplitude \( \phi(t) \), while the coefficient \( k \) is determined by the design of the instrument.

The actual signal is an additive mixture of the useful signal \( U(t) \) and the noise \( \xi(t) \):

\[
x(t) = U(t) + \xi(t).
\]

We will assume that we have the sample \( \{x_i\} \) \((i = 1, 2, \ldots, m)\) of the signal \( x(t) \) at equidistant moments of time \( t_i \) with the interval \( \Delta t \). Here, \( U_0(t), \phi(t) \) are deterministic functions of time, \( \varphi_0 = 0 \), and \( \alpha \) is an unknown parameter.

As the model of the noise in (2), we take white Gaussian noise \( n_0(t) \) with a one-sided spectral energy distribution. We will write the joint probability density function for values of the random variable \( n_{0i} \), \( i = 1, 2, \ldots, m \). These values are normally distributed and independent and have the following statistical characteristics:

\[
M[n_{0i}] = 0; \quad D_{0i} = M[n_{0i}^2] = \frac{N_0}{2\Delta t}; \quad M[n_{0i}, n_{0j}] = \delta_{ij}.
\]

Here, the \( m \)-dimensional noise distribution can be represented in the form

\[
\Pi_m(n_{01}, \ldots, n_{0m}) = \prod_{i=1}^{m} p_i(n_{0i}) = \left( \frac{N_0}{\Delta t} \right)^{-m/2} \exp\left\{-\frac{1}{N_0} \sum_{i=1}^{m} n_{0i}^2 \Delta t\right\}.
\]

We will construct the similitude functional [2] on the basis of the parameter \( \alpha \). With allowance for (2), we do this by changing over to a variable \( \alpha \) in (3):

The maximum of similitude functional (4) will be seen at the point $\alpha = \hat{\alpha}$, which is the most likely value of the parameter being estimated and is found on the basis of the condition

$$\frac{\partial \ln L(\alpha)}{\partial \alpha} \bigg|_{\alpha = \hat{\alpha}} = 0.$$  

(5)

It is known that when the signal/noise ratio is high and the sample is large, estimates of constant parameters made by the method of maximum similitude are asymptotically stable, effective, and normally distributed [2]. Thus, Eqs. (4) and (5) constitute an asymptotically optimum algorithm for evaluating the amplitude of vibrations on the basis of the signal of an LDA against a background of additive white Gaussian noise.

We used statistical modeling to check the efficiency of the algorithm proposed here. The following functions were used as model signals in calculations performed by means of algorithm (4), (5)

$$z(t) = \sum_{k=1}^{4} A_k \cos(k\Omega t) \quad \text{and} \quad U_0(t) = 1 + m \cos(\Omega t).$$

(6)

Figure 1 shows the results of the calculations. The noise/signal ratio is plotted off the x-axis and the relative error of the estimate of the parameter $\alpha$ is plotted off the y-axis. It follows from the figure that the algorithm (4), (5) makes it possible to evaluate the amplitude of vibrations with an error no greater than the sampling interval for $\alpha$ in (4) when the value of the signal/noise ratio at the input is at least 5.

Now let us assume that in (1) $\gamma = 1$, $U_0(t)$ is a deterministic function, $q_0 = 0$, and $z(t)$ is the unknown law of motion of the object. We will regard $z(t)$ as the parameter $z$ changing over time.

We will form the optimum estimate $\hat{z}$ of the parameter $z$ on the basis of the bias approach. In this case, the optimality criterion is linked with the so-called loss function [2]. The bias estimate obtained with a quadratic loss function is optimum when judged by the criterion of the minimum standard deviation and is the center of gravity of the a posteriori probability density function

$$\hat{z}_{pd} = \int z q_p(z \mid q_0) dz.$$

(7)