The behavior of the spectral density of the thermal noise of laser gravity antennas fastened by an elastic clip with dissipative losses in the mounting is investigated, using a smooth perturbations method based on the fluctuation-dissipation theorem.

A Fabry–Perot interferometer, formed by mirrors \( m_1 = m_2 = M \), and suspended from thin straight rods (Fig. 1), is a one-dimensional model of a free-mass laser-interference antenna. A weak gravitational wave acting on the interferometer changes the distance \( L(t) \) between the mirrors,

\[
L(t) = L_0 \left( 1 + \frac{h(t)}{2} \right),
\]

where \( h(t) \) are dimensionless variations of the metric and \( L_0 = 4 \times 10^{-3} \) m is the interferometer base.

The influence of the suspension thermal noise must be taken into account when detecting pulses ("bursts") of gravitational radiation of amplitude \( h \leq 10^{-21} \). One source of thermal noise is the internal resistance of the suspension material \([1, 2]\), which has an elastic modulus \( E \) that is a complex function of the frequency \( \omega \):

\[
E = E(\omega) = E_0(\omega) e^{i\omega t}.
\]

Dissipative losses in the mounting may be another major source of thermal noise. The simplest model of elastic attachment, ensuring that the suspension point \( x = 0 \) can be displaced further, is shown in Fig. 2. The compressive (tensile) stiffness \( K \) of the support depends on the specific design of the suspension. If dissipative loss occurs, particularly hysteresis loss which is characteristic of the inelastic resistance of the material, the stiffness \( K \) is a complex function of the frequency:

\[
K = K(\omega) = K_0(\omega) e^{i\omega t}.
\]

The frequency dependence (2) of the support stiffness \( K \) is entirely analogous to the dependence (1) of the elastic modulus of the suspension material and is due to the same physical effects.

The goal here is to calculate the spectral density \( S(\omega) \) of the thermal noise of the mirror when the upper end of the rod \( x = 0 \) is fastened elastically (the mirror is considered as a concentrated mass \( m \) at the lower end of the rod \( x = l \)). The internal resistance of the rod material is ignored: \( \text{Im} E(\omega) = 0 \).

The transverse oscillations \( Y(x, t) \) of a thin straight rod are described by a biharmonic wave equation \([2, 3]\). In the frequency range that equation can be written as

\[
-\nu \ddot{Y}^{(4)}(\xi) - \left( 1 + a \right) \ddot{Y}^{(2)}(\xi) + Y(\xi) = 0, \quad 0 < \xi < \frac{\pi}{\lambda} l
\]
In Eq. (3) we have used the notation

\[
\epsilon = \lambda^2 E / P; \quad \alpha = I_0 \omega^2 / P; \quad \lambda = \omega / \nu; \quad \nu = \sqrt{P \mu_1}; \quad \xi = \lambda x; \quad Y^{(1)}(\xi) = d^2 \tilde{Y} / d \xi^2.
\]

where \( E \) is the elastic modulus of the suspension material, \( I \) is the cross-sectional moment of inertia of the rod with respect to the central axis of the cross section perpendicular to the plane of vibrations, \( I_0 \) is the moment of inertia of the rod length with respect to the axis perpendicular to the plane of vibrations, \( P = mg \) is the tensile force applied to the lower end of the rod, \( g \) is the free fall acceleration, \( \mu_1 \) is the mass of a unit length of rod, and \( \tilde{Y} = \tilde{Y}(x, \omega) \leftrightarrow Y(x, t) \) (the argument is omitted henceforth in the analysis).

The biharmonic equation (3) contains the small parameter \( \epsilon \) in the higher derivative. The limiting case \( \epsilon = 0 \) corresponds to the string approximation. For a small but nonzero \( \epsilon \) we look for the solution of the equation in the form

\[
Y(x) = AC \cos \nu x - B \sin \nu x, \quad \nu < \nu < \nu_0;
\]

\[
\nu = 1 - (\epsilon + a)/2.
\]

where \( A \) and \( B \) are the constants of integration, which are determined from the boundary conditions [2, 3]. For a "string" those boundary conditions can be written as

\[
P \lambda \tilde{Y}^{(1)}(0) = K \tilde{Y}(0), \quad P \lambda \tilde{Y}^{(1)}(\xi_0) - m \omega^2 \tilde{Y}(\xi_0) = \tilde{F},
\]

where \( \lambda \tilde{Y}^{(1)}(\xi) \) is the shearing force [3] in the string approximation \( \epsilon \to 0 \). \( \xi_0 = \lambda \xi, \tilde{F} = \tilde{F}(\omega) \leftrightarrow F(t) \), and \( F(t) \) is a concentrated force applied to the concentrated mass \( m \).