We show that the characteristic geometric parameters of a local velocity transducer for a flow with an internal magnetic-field source can be determined by analyzing an electromagnetic flow meter of conventional design by using the method of conformal mappings from the theory of functions of a complex variable. We show that this method can be used to obtain an electromagnetic transducer with optimal parameters for measurement of both speed and flow rates of liquids in large pipes and open channels.

Electromagnetic flow meters are widely used to measure flows of liquids and thermal energy in pipes of diameters ranging from 8-10 mm to 3,000-4,000 mm. There are two basic designs for electromagnetic flow meters: those with magnetic fields permeating the entire cross-section of the channel, and those with magnetic fields that penetrate only the boundary of the flow. The first design is well known, has been studied, and widely used. This design has become conventional, although instruments using this design are used primarily only for measurement flows in pipes with small and medium-sized diameters (no more than 300-600 mm), since, as the diameter of the channel increases, the cost and size of the instrument increase sharply.

The second design has not been studied thoroughly, although it certainly shows promise for measurement of flows in large-diameter pipes. Such an instrument includes one or more electromagnetic local-speed transducers inside a pipe of diameter from 400 to 4,000 mm. The flow rate is measured with the area-velocity method. We will discuss problems on optimization of the design of transducers for local flow speeds that can be used to measure the flow rates in large-diameter pipes.

The solutions to problems on the distribution of electric fields in measured media for electromagnetic flow meters and local velocity transducers differ only in the boundary conditions, which are determined by the geometric configurations of the electrode surfaces and insulating walls in contact with the measured flow. These surfaces divide space into two characteristic regions, the inside and outside, and while a transducer for the flow inside a region filled with liquid has its inductor located outside, for a velocity transducer, the measured medium fills the outside region, and the inductor is located inside. This circumstance makes it possible to establish analogies and differences for phenomena related to flow and velocity transducers; it also makes it possible to use them to design optimal transducers.

As we know, the best electromagnetic flow meters have rectangular channels. Maximum sensitivity for such flow meters is provided by orthogonality of the magnetic field, the flow velocity, and the lines of flux between the electrodes everywhere in the working cross-section. The surfaces of the electrodes are located along the equipotential surfaces of the induced electric field. When the magnetic field is uniform in the rectangular channel of a flow meter, the reading does not depend on the velocity profile of the flow. All of this shows that the problem of optimizing a velocity transducer can be solved by analyzing a flow meter with a rectangular channel [1].

We locate the cross-section of a rectangular channel in the complex plane $S(\xi, \nu)$ so that the $\xi$ axis passes through the middle of the electrodes and the $\nu$ axis is equidistant from them (Fig. 1). The contour bounding the interior region of a rectangular flow meter consists of line segments parallel to the coordinate axes $\xi$ and $\nu$. We can use an inversion to map the interior region of a flow transducer onto the region that is exterior to the rectangular contour. In the $Z(X,Y)$ plane we reflect the line segments into arcs of circles that are tangent to the coordinate axes $X$ and $Y$ at the coordinate origin; we map the electrodes onto arcs of circles that are tangent to the $Y$ axis, and we map the insulating walls onto arcs that are tangent to the $x$ axis.

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The contour bounded by the arcs in the $Z(X,Y)$ can be treated as the cross-section of a velocity transducer. Now the space outside the contour of the velocity transducer in the plane $Z(X,Y)$ is reflected onto the space inside the channel contour in the $S(\xi,\nu)$ plane, and conversely. After reflection, the space occupied by the coils exciting the magnetic field in the $Z(X,Y)$ plane occupies part of the space bounded by the circles tangent to the $Y$ axis at the coordinate origin. The part of the space in the $S(\xi,\nu)$ plane that is adjacent to the insulating walls, which usually contains the inductor poles, is reflected onto the region inside the contour of the velocity transducer in the $Z(X,Y)$ plane, where the poles of radius $R$ of the inductor are located.

It is not difficult to see that in the $Z(X,Y)$ plane, the vectors of magnetic induction $B$ and current $I$ between the electrodes are orthogonal throughout the space outside the contour of the velocity transducer, just as they are in the $S(\xi,\nu)$ plane in the channel of a flow transducer. Thus, to construct an output signal, we use all of the space we have considered, which is the distinguishing property of the velocity transducer under discussion. Using the method of conformal mapping, we can express a weighting function $W$ and the magnetic induction of the field $B$ in the transducer as

$$W = \frac{R}{\rho^2}, \quad (1)$$

$$B = \frac{\mu_0 R I_0}{\rho^2}, \quad (2)$$

where $I_0$ is an ampere turn of the excitation coil, $\mu_0$ is the magnetic permeability of the measured medium, $R$ is the radius of the inductor, and $\rho$ is the current radius.

Analysis of expressions (1) and (2) shows that lines of equal absolute values for $W$ and $B$ form a family of concentric circles. The rate of decrease in the absolute values of $W$ and $B$ is the same and inversely proportional to the square of the distance from the point under consideration to the coordinate origin (which coincides with the center of the cross-section of velocity transducer). The fact that a flow meter with a rectangular channel is insensitive to the flow profile is transformed, for a velocity transducer, to the fact that it is insensitive to the velocity distribution relative to the coordinate polar angle $\theta$.

In order to determine the sensitivity of the instrument to flow velocity, we assume that $v = v_z = \text{const}$, $v_x = v_y = 0$.

In this case the potential difference between the electrodes in the velocity transducer is

$$U = \bar{v} \int_S [W \times B] \, dS, \quad (3)$$

where $S$ is the region occupied by the fluid.

Denoting the measurement sensitivity by $S_m = U/v$ and using expressions (1), (2), and (3), we obtain