NOISE IMMUNITY AND SPEED OF FREQUENCY MEASUREMENTS OF SHORT HARMONIC SIGNALS

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Statistical simulation methods are used to investigate the noise immunity and speed of response of five methods of measuring the frequency of short harmonic signals and noise.

In the processing of subsonic harmonic waves or radio-frequency pulses with a few periods (e.g., in Doppler velocity meters) there is a need of measuring signal frequency during a short time interval equal to 0.5–2 oscillation periods. The investigated harmonic signal is assumed to be received on the background of normal white noise $\xi(t)$ with zero average value and a standard deviation (SD) $\sigma$, i.e., the frequency $\omega$ must be evaluated from a short sample of signal values:

$$u(t) = a\sin(\omega t) + b\cos(\omega t) + \xi(t),$$

where $A$ is the harmonic signal amplitude.

Several different algorithms for estimating the signal frequency have been proposed during the past few years [1–5].

1. The “optimal” method [1]. The signal frequency $\omega$ is found from the condition of the global maximum of an objective function of the form

$$\Gamma(\omega) = \frac{U_c^2}{\lambda_1} + \frac{U_s^2}{\lambda_2},$$

where $U_c = \frac{1}{K} \sum_{i=1}^{K} u_i \cos(\omega t_i)$; $U_s = \frac{1}{K} \sum_{i=1}^{K} u_i \sin(\omega t_i)$;

$$\lambda_1 = \frac{1}{2} \left( 1 + \frac{\sin(\omega T_s)}{\omega T_s} \right); \quad \lambda_2 = \frac{1}{2} \left( 1 - \frac{\sin(\omega T_s)}{\omega T_s} \right);$$

$T_s$ is the time during which the instantaneous signal values are sampled with an analog-digital converter (ADC), $T_s = K\Delta t$, $\Delta t$ is the ADC sampling period, and $u_i$ is the instantaneous signal value at the time $t$.

2. “Operational” method [2]. The frequency is determined as $\omega = \omega_0 x$, where $\omega_0$ is the center of the range of possible frequencies, and $x$ is found from the equation

$$\frac{1}{\lambda_1} U_c^2 x_1(x) + \frac{1}{\lambda_2} U_s^2 x_2(x) - \frac{2}{T_s} \left( \frac{U_c U_s}{\lambda_1} + \frac{U_s U_c}{\lambda_2} \right) = 0;$$

$$x_1(x) = \frac{1}{\alpha_0} \left( \frac{\sin(x-1) \alpha_0}{(x-1)^2} - \frac{\sin(x+1) \alpha_0}{(x+1)^2} \right).$$

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Fig. 1. Relative error frequency measurement error as a function of the ratio $T_s/T$ and noise level $S$ (dotted curve — method 2, solid curve — method 3).

\[-\frac{\cos (x-1) \alpha_0}{x-1} - \frac{\cos (x+1) \alpha_0}{x+1}\times\]

\[\times \left[\frac{\sin (x-1) \alpha_0}{x-1} + \frac{\sin (x+1) \alpha_0}{x+1}\right]^{-1}\]

\[\chi_2 (x) = \frac{1}{\alpha_0} \left(\frac{\sin (x-1) \alpha_0}{(x-1)^2} + \frac{\sin (x+1) \alpha_0}{(x+1)^2}\right) -\]

\[-\frac{\cos (x-1) \alpha_0}{x-1} - \frac{\cos (x+1) \alpha_0}{x+1}\times\]

\[\times \left[\frac{\sin (x-1) \alpha_0}{x-1} + \frac{\sin (x+1) \alpha_0}{x+1}\right]^{-1}\]

\[U_c = \frac{1}{K} \sum_{i=1}^{K} u_i \cos (\omega_0 t_i); \quad U_s = \frac{1}{K} \sum_{i=1}^{K} u_i \sin (\omega_0 t_i);\]

\[g_c = \frac{1}{K} \sum_{i=1}^{K} u_i t_i \cos (\omega_0 t_i); \quad g_s = \frac{1}{K} \sum_{i=1}^{K} u_i t_i \sin (\omega_0 t_i);\]