ADDITIVE POLARIZATION PROBLEM IN OPTIMAL MEASUREMENT SYSTEMS

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We compute the polarization states of superpositions of arbitrary numbers of radiations. The degree of coherence and both the amount and type of polarization are arbitrary. The mathematical apparatus of coherence matrices is used. The special case of coherent superposition of linear and circular polarizations is investigated.

The problem of additive polarization is to determine the polarization state of the superposition of an arbitrary number of radiations when the polarization state of each component in the composite is known. This problem is extremely pressing in optical measurement technology. It appears, for example, during analysis of optical roentgenography and is important when we must account for multiple reflections that cause the polarization to change each time a reflection occurs. Finally, the additive polarization problem is of cardinal value for determining the parameters for interference instruments that separate different channels by polarization. The problem is trivial for incoherent superpositions. However, it is rather difficult in the case of mutual coherence of composite radiation and an arbitrary degree of polarization in each component.

We begin by considering the problem in its simplest form, where we deal with the superposition of only two components. Suppose that there are two radiation components that are each (i-th) characterized by specific polarization characteristics: intensity $I_i$, degree of polarization $P_i$, azimuth $\theta_i$, and ellipticity $\epsilon_i$ of the polarization ellipse. We correlate the added light beams by using the complex coherence

$$g_{ik} = |g_{ik}| \exp(j\delta_{ik}).$$

(1)

where $|g_{ik}|$ is the coherence of the radiation under consideration; $\delta_{ik}$ is the phase difference between the $x$-th components of the polarization components of the radiation.

It is most convenient to use the apparatus of coherence matrices [1, 2] to solve the general additive polarization problem. The method is based on computation of the resultant Cartesian coherence matrix

$$J_x = \left( \begin{array}{c} J_{xx}^n + J_{yy}^n \\ J_{xy}^n \end{array} \right) = \left( \begin{array}{c} <E_{xx}^n E_{xx}^n> + <E_{xy}^n E_{xy}^n> \\ <E_{xy}^n E_{xx}^n> + <E_{xy}^n E_{xy}^n> \end{array} \right).$$

(2)

where $E_x$ and $E_y$ are the Cartesian components of the total radiation; $< >$ denotes averaging over some time interval that is much larger than the time of mutual coherence of the Cartesian components, but less than the reaction time of the photodetector.

To write out the coherence matrix for the total radiation (2) in explicit form, we must account for the fact that each Cartesian component of each radiation component contains polarized parts $E_{x,y,i}^p$ and unpolarized parts $E_{x,y,i}^n$. The unpolarized component of radiation is a random function of time that takes positive and negative values with equal probability over the averaging interval, so the average product of this function and any other independent function is zero. For the purpose of averaging the products that appear in (2), we have the following natural and obvious relations:

$$<E_{xx}^n E_{xx}^n> = <E_{xy}^n E_{xy}^n> = <E_{xx}^n E_{xy}^n> = <E_{xy}^n E_{xx}^n> = 0,$$

$$<E_{xy}^n E_{xy}^n> = |g_{ik}| \exp(j\delta_{ik}) A_{ik}^n A_{ik}^n.$$
where the $A_{x,y,i,k}^p$ are the amplitudes of the Cartesian components of the polarized components, and the $A_{x,y,i,k}^u$ are the same for the unpolarized components; $\Phi_{ik}$ is the phase difference between the Cartesian component of the polarized components of each radiation component. These quantities are associated with initial parameters by known relations [2]:

$A_{x,y,i}^p = \sqrt{P_i(1 + \cos 2\theta_i \cos 2\phi_i)}$, 

$A_{x,y,i}^u = \sqrt{P_i(1 - \cos 2\theta_i \cos 2\phi_i)}$, 

$\sin \Phi_i = \frac{\sin 2\phi_i}{\sqrt{1 - \cos^2 2\theta_i \cos^2 2\phi_i}}$, 

$\cos \Phi_i = \frac{\sin 2\theta_i}{\sqrt{1 - \cos^2 2\theta_i \cos^2 2\phi_i}}$, 

$A_{i}^u = A_{i}^p = \frac{1}{\sqrt{2}} (1 - P_i)$

(3)

We can now compute the elements of the resultant coherence matrix:

$J_{xx} = <E_x E_x^* > = < (E_{x1}^p + E_{x1}^u + E_{x2}^p + E_{x2}^u) > < (E_{x1}^* + + E_{x2}^*) > = (A_{x1}^p)^2 + |g_{12}| \exp(j\Phi_{12}) A_{x1}^p A_{x2}^p +$ 

$+ |g_{12}| \exp(j\Phi_{12}) A_{x1}^u A_{x2}^u + (A_{x2}^p)^2 + (A_{x2}^u)^2 +$ 

$+ |g_{12}| \exp(j\Phi_{12}) A_{x1}^p A_{x2}^u + |g_{12}| \exp(j\Phi_{12}) A_{x1}^u A_{x2}^p + (A_{x2}^p)^2$ 

$J_{yy} = <E_y E_y^* > = < (E_{y1}^p + E_{y1}^u + E_{y2}^p + E_{y2}^u) > < (E_{y1}^* + + E_{y2}^*) > = (A_{y1}^p)^2 + |g_{12}| \exp(j\Phi_{12}) A_{y1}^p A_{y2}^p +$ 

$+ |g_{12}| \exp(j\Phi_{12}) A_{y1}^u A_{y2}^u + (A_{y2}^p)^2 + (A_{y2}^u)^2 +$ 

$+ |g_{12}| \exp(j\Phi_{12}) A_{y1}^p A_{y2}^u + |g_{12}| \exp(j\Phi_{12}) A_{y1}^u A_{y2}^p + (A_{y2}^p)^2$ 

$J_{xx} = <E_x E_x^* > = < (E_{x1}^p + E_{x1}^u + E_{x2}^p + E_{x2}^u) > < (E_{x1}^* + + E_{x2}^*) > = (A_{x1}^p)^2 + |g_{12}| \exp(-j\Phi_{12}) A_{x1}^p A_{x2}^p +$ 

$+ |g_{12}| \exp(-j\Phi_{12}) A_{x1}^u A_{x2}^u + (A_{x2}^p)^2 + (A_{x2}^u)^2 +$ 

$+ |g_{12}| \exp(-j\Phi_{12}) A_{x1}^p A_{x2}^u + |g_{12}| \exp(-j\Phi_{12}) A_{x1}^u A_{x2}^p + (A_{x2}^p)^2$ 

$J_{yy} = <E_y E_y^* > = < (E_{y1}^p + E_{y1}^u + E_{y2}^p + E_{y2}^u) > < (E_{y1}^* + + E_{y2}^*) > = (A_{y1}^p)^2 + |g_{12}| \exp(-j\Phi_{12}) A_{y1}^p A_{y2}^p +$ 

$+ |g_{12}| \exp(-j\Phi_{12}) A_{y1}^u A_{y2}^u + (A_{y2}^p)^2 + (A_{y2}^u)^2 +$ 

$+ |g_{12}| \exp(-j\Phi_{12}) A_{y1}^p A_{y2}^u + |g_{12}| \exp(-j\Phi_{12}) A_{y1}^u A_{y2}^p + (A_{y2}^p)^2$