ANALYSIS OF THE ELLIPSE OF POLARIZATION 
BY THE SENARMONT METHOD AS THE PHASE 
DIFFERENCE VARIES IN THE RANGE 0-2π

V. I. Chmyrev and V. M. Skorikov

An analysis of the Senarmont method is performed using Müllner matrices and Stokes vectors. It is proved that the traditional understanding of the Senarmont method is applicable in the case of phase differences not exceeding one-half wave. For measurements of phase differences in the range 0-2π, a preliminary identification of the rapid and slow axes of the phase plate is required. A method is presented by means of which it is possible to distinguish the axes on the basis of the direction of the rotation of the resultant plane oscillations in the case of continuous variation of the phase differences.

All the different methods of measurement of natural or induced elliptical polarization of phase plates basically come to two methods, an absolute method in which the intensity of a previously transmitted light wave must be measured, and a null method in which an unknown phase difference is complemented until the transmission of the system has been reduced to zero. It is clear that null methods are more preferable, since in this case the fluctuations in the transmission of the measurement channel have little effect on the precision of the phase difference measurement.

The Senarmont method, a variety of the null method that possesses all of its advantages, has rarely been applied in practical studies by researchers, due not only to objective factors, such as the absence of achromatic quarter-wave apparatus, but also because none of the existing manuals have been written for conditions involving arbitrary phase differences. The most well-known description of the Senarmont method may be found in the widely distributed monograph [1]. In this monograph it is asserted that, using the Senarmont method, it is possible to measure phase differences within a single wavelength or the excess above a integral number of waves, though, as will be demonstrated below, the method is suitable only for phase differences not exceeding π. In attempts at measuring phase differences greater than π by means of the Senarmont method, contradictory results may appear which cannot be explained by the existing manuals. Is this the reason why the monograph [1] recommended the Senarmont method for the measurement of small phase differences? How widespread is this misunderstanding is demonstrated by one of the most reliable monographs [2], which in its description of the method does not say a word of the need for preliminary identification of the principal axes of the phase plate even though in this case it is not possible to measure phase differences exceeding π. In the present article we will analyze the necessary formalism of the Senarmont method with exhaustive completeness using Müllner matrices and Stokes vectors. This will enable us to reveal all the advantages of the method and, without any question, help in bringing it into increasingly more widespread use.

Henceforth, we will agree that light propagates in the positive direction of the Z-axis to the observer, and that the polarizers, specimen, and quarter-wave plate are situated in the XY plane (Fig. 1). The line of intersection formed by the plane of oscillation of the polarizer and the XY plane is adopted as the base line of the frame P. We will measure off from this line all angles θ in the positive direction (counter-clockwise). For the analyzer the same line will be denoted A. In the initial position P and A are mutually perpendicular, the principal axes of the specimen n' and n" are at angles θ = ±45° to the base line P, and the rapid axis of the quarter-wave plate P(λ/4) is parallel to P. Since we do not know which of the axes of the specimen n' or n" is the rapid axis, the calculation must be performed for both cases.
Fig. 1. Schematic of calculation. $S_j$) Stokes vectors; $M_j$) Müller matrices, with
A) analyzer; P) polarizer; S) specimen; $\lambda/4$) quarter-wave plate.

I. $P$ orientation (Fig. 2a).

Let $n' = n_\parallel$ be the rapid axis and $n'' = n_\perp$ the slow axis. The common transit of the system by a beam of natural light of
intensity $I_0$ may be described thus:

$$S_5 = M_4 M_3 M_2 M_1 S_1,$$

where $S_1$ is the Stokes vector of the incoming natural beam of light, which is characterized by the Stokes column vector
$S_1 = [I_0, 0, 0, 0]$; $S_5$ is the Stokes vector of the outgoing beam; $M_4, M_3, M_2,$ and $M_1$ are the Müller matrices of the analyzer, $\lambda/4$
plate, specimen, and polarizer, respectively (all the initial matrices are from [3, 4]).

For the polarizer matrix with $\theta = 0^\circ$,

$$S_2 = M_1 S_1 = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} I_0 \\ 1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} I_0 \\ 1 \\ 2 \\ 0 \end{pmatrix}.$$

Following the specimen the state of polarization is described by the Stokes vector

$$S_3 = M_2 S_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c & 0 & -s \\ 0 & 0 & 1 & 0 \\ 0 & s & 0 & c \end{pmatrix} \begin{pmatrix} 1 \\ I_0 \\ 1 \\ 2 \\ 0 \end{pmatrix}.$$

where $M_2$ is the matrix of the phase plate, which introduces a phase difference $\delta$ from the rapid axis directed at an angle $\theta = \pi/4$
to the base line $P$; $c = \cos \delta$, $s = \sin \delta$.

Once the light has traversed the quarter-wave plate, the rapid axis of which is directed at an angle $\theta = 0^\circ$, we obtain