PHYSICOCHEMICAL MEASUREMENTS

DETERMINING FLOW COMPONENT CONCENTRATIONS
FROM THE REAL AND IMAGINARY PARTS OF THE
DIELECTRIC CONSTANT MEASURED IN THREE
ORTHOGONAL DIRECTIONS

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A method is presented for measuring the concentrations of flow components from their static complex
dielectric constants. The initial equations are derived, and also a solution to the inverse treatment by a
nonconvex programming method. A description is given of the measurement system.

It is difficult to overestimate the importance of determining the compositions of mixtures flowing in closed pipes.
There is a wide range of physical methods (optical, acoustic, and radio-frequency ones), but only the latter are suitable
because the characteristic scales of the flow inhomogeneities are much less than the corresponding wavelengths, while the metal
walls give closely defined boundary conditions, in contrast to the conditions in acoustic methods.

The method involves measuring the real and imaginary parts of the dielectric constant in three mutually perpendicular
directions by reference to the measured resonant frequencies and quality factors of the corresponding cavities, where one uti-

Electrostatic Model. We use a coordinate system related to the symmetry axes of the pipeline and employ the theorem
on the mean \[ I \] to write the equation for the electrical induction as

\[ \int dV E_k(r) [\varepsilon(r) - \varepsilon_k] = 0. \tag{1} \]

Here we have used the fact that although the components of the mixture are isotropic, the mixture itself is anisotropic
because of deviations from symmetry due to the direction of the flow velocity and that of the field of gravity.

We split up the volume into finite parts and group them by components (subscript \( i \)) and suspended-particle shapes
(subscript \( j \)) on the assumption that the fields within the identical particles \( E_{k,ij} \) are identical:

\[ \sum_{i,j} [E_{k,ij} f_i s_j (\varepsilon_i - \varepsilon_k)] = 0, \tag{2} \]

in which \( f_i \) is the volume fraction of component \( i \) and \( s_j \) is the volume fraction of shape \( j \):

\[ \sum f_i = 1; \tag{3} \]
We use expressions for the fields within ellipsoids [2] to get from the single vector equation (2) three each scalar equations for the real and imaginary parts:

\[ \text{Re} \sum_{l,j} \frac{Re_{ik}}{1 + \Re_{ik} n_{jk}} f_l s_j = 0, \]

\[ \text{Im} \sum_{l,j} \frac{Im_{ik}}{1 + \Im_{ik} n_{jk}} f_l s_j = 0, \]

in which \( g_{ik} = (\varepsilon_i / \varepsilon_k) - 1 \), with \( \varepsilon_k \) the dielectric constant of the mixture \( \varepsilon \) as measured in direction \( k \), \( n_{jk} = 1/3 \) for a sphere, \( n_{jk} = 0 \) for a disk, planar layer, tube in the direction of the axis or in the plane of a disk, \( n_{jk} = 1/2 \) for a tube perpendicular to the axis, and \( n_{jk} = 1 \) for a disk perpendicular to its plane.

Equations (5) reflect the actual system approximately. The errors in the model are due to fluctuations in the mean field in the volumes around the suspended particles on account of correlations between the fields of adjacent particles.

Out of the standard models, (5) is the most likely and includes as special cases the models of [2, 3] and also completely stratified systems. Finally, the range of application for that model should be checked in model experiments.

When one considers the inversion of (5) with the use of the (3) and (4) normalization conditions for a completely anisotropic system showing absorption, i.e., when all six equations in (5) are distinct, one can handle the task in which the sum of the number of components and the number of forms is equal to eight. The minimum number of forms is 3-4 (spheres, disks stratified in the field of gravity, disks stratified in the flow direction, and possibly tubes stratified in the flow direction), so one can solve the system for 5-4 components respectively. If there is no absorption or it is not measured, one can solve a system for two components, but not always then.

The above implies that detailed measurements must incorporate the flow symmetry and the disposition of it with respect to the forces acting in order to minimize the number of shapes involved.

Model calculations show that this model cannot be replaced by a simpler anisotropy model, e.g., one containing identical ellipsoids with longitudinal orientations for their axes.

**Direct Treatment of Dielectric Constant Determination.** The first task is to determine the dielectric constants of the mixture in the various directions, as the solution enables one to check the errors and reliability in the definition of the model by means of special experimental model mixtures.

The solution to (5) in terms of the six unknowns \( \text{Re} \varepsilon_k \) and \( \text{Im} \varepsilon_k \) is obtained from six independent equations, since (5) splits up in that way because of the conditions \( \text{Im} \varepsilon_k \ll \text{Re} \varepsilon_k \) and \( \text{Im} \varepsilon_l \ll \text{Re} \varepsilon_l \).

Each of these equations is solved numerically by a weighting method analogous to the operating algorithm for analog digital converters. In 20 steps, it is possible to determine the unknowns with an error of \( 10^{-7} \).

**Inverse Treatment: Determining Component Concentrations.** In writing software for solving the inverse treatment for (5), we initially used an algorithm for minimizing a quadratic functional of (5) by convex programming methods employing projection of the antigradient on the permissible region of values for the vectors \( f \) and \( s \), which have sign-positive components. However, the resulting solutions were highly smoothed and unstable. An analysis indicated that the solution must be obtained by minimizing the sum of the absolute values in (5). That solution was found to be completely satisfactory.

Then the algorithm for the inverse treatment of (5) involves minimizing

\[ w(f,s) = \sum_{2k} \left[ \text{fabs} \left( \frac{\text{Re} \sum_{l,s} \frac{Re_{ik}}{1 + \Re_{ik} n_{jk}} f_l s_j}{\text{Im} \sum_{l,s} \frac{Im_{ik}}{1 + \Im_{ik} n_{jk}} f_l s_j} \right) \right] \]

The initial point in the iteration was taken as distributions uniform in the components and particle shapes:

\[ f^0 = 1/L; \quad s^0 = 1/K. \]