ANALYSIS OF PIEZOELECTRIC ULTRASONIC RECEIVERS
BASED ON THE EQUATIONS OF ELECTROELASTICITY WHEN
THE SENSITIVE ELEMENT IS IN A STATE OF BULK STRESS

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A new method of analyzing ultrasonic receiving transducers, based on a calculation of the electroelasticity of
a piezoelectric element when the sensitive element is in a state of bulk stress is presented. The method enables
the structural parameters of pickups to be optimized, taking into account the input circuits of the amplifier-
converter apparatus.

There are a large number of publications devoted to different aspects of the problem of analyzing the construction and
manufacture of piezoelectric transducers and of matching the impedance of the transducers to the channel [1-5]. The majority
of the published papers are based on simplified engineering approaches when investigating complex processes in piezoelectric
materials. Nevertheless, in view of the intensive development of computers, more rigorous mathematical methods of analyzing
the combined fields of piezoelectric elements within the framework of the mechanics of deformed piezoelectric materials have
become possible. In particular, different versions of the theory of plates and shells have been developed, devoted to a successive
description of the theory of wave processes in piezoelectric materials [4]. On the basis of this, we have developed a new
method, which involves calculating the electroelasticity of the piezoelectric element when the sensitive element is in a state of
bulk stress, taking into account the reflection and transmission of a body wave through the structural elements of the transducer.
The main difference between this method and others is the fact that it takes into account all the components of the strain tensor
of the piezoelectric element, including the components which, for specific boundary conditions, may be equal to zero. We
assume here that when there is no strain in the piezoelectric element in a specific direction there is a stationary mode of
oscillations which may have a considerable effect on the results of an analysis of an acoustic transducer. When all the dimensions
of the piezoelectric element are commensurable with the wavelength of the acoustic signal, it is extremely difficult to isolate the
predominant mode of oscillation. This problem can be eliminated, when analyzing piezoelectric transducers, by determining
the characteristics of the system of equations of electroelasticity when the sensitive element is in a state of bulk stress, when
the equations are largely set up for all the forces acting on the faces of the piezoelectric element. The parameters of the input
circuits of the amplifier-transducer are also taken into account; this is extremely important for systems for monitoring and for
the diagnostics of electrical equipment, including long cable lines. Here we must emphasize that the class of problems that can
be solved is limited to those with weak perturbations, and second-order effects are ignored.

The proposed method enables one to optimize the constructional-component parameters of piezoelectric ultrasonic
receivers. This can be illustrated using the example of an analysis of acoustic-emission transducers, constructed using a sensitive
element in the form of a short cylinder and a ring placed on a protector in the form of a thin plate.

The equations of state for a piezoceramic, polarized in the z direction, have the form of the equations of state of a
piezoelectric crystal of the hexagonal class of symmetry 6 mm. In a cylindrical system of coordinates r, θ, z, the equations
connecting the stresses \( \sigma \), strains \( s \), the electric field strength \( E_z \), the potential difference between the electrodes of the
piezoelectric element \( U \), and the piezoelectric constants of the ceramic \( \varepsilon_{33} \), \( c_{ij} \), and \( e_{ij} \), take the form

\[
\begin{align*}
\sigma_r &= c_{11}^E s_r + c_{12}^E s_\theta + c_{13}^E s_z - e_{31} E_z ; \\
\sigma_\theta &= c_{12}^E s_r + c_{11}^E s_\theta + c_{13}^E s_z - e_{31} E_z ; \\
\sigma_z &= c_{13}^E (s_r + s_\theta) + c_{33}^E s_z - e_{33} E_z ; \\
D_z &= \varepsilon_{33}^E E_z + e_{31} (s_r + s_\theta) + e_{33} s_z ; \\
s_r &= \frac{\partial u_r}{\partial r} ; s_\theta = \frac{1}{r} \frac{\partial u_\theta}{\partial r} ; s_z = \frac{\partial u_z}{\partial z} ; E_z = - \frac{\partial U}{\partial z} ,
\end{align*}
\]

where \( u \) is a component of the displacement tensor.

For normal incidence of a plane longitudinal wave on the end surface of the piezoceramic cylinder, three modes of oscillation propagate in it: thickness, radial and circular.

The solution of the system of equations (1) for the nodes of oscillation must be sought in the form of standing waves of the displacement \( u \):

\[
\begin{align*}
u_z &= (A \sin(k_z z) + B \cos(k_z z)) e^{i \omega t} , \\
u_r &= (C \sin(k_r r) + D \cos(k_r r)) e^{i \omega t}, \\
u_\theta &= E e^{i \omega t},
\end{align*}
\]

where \( A, B, C, D, \) and \( E \) are coefficients which depend on the boundary conditions.

Here, in the axisymmetrical case considered, the circular mode of oscillations of the cylinder has a stationary solution.

When a plane longitudinal wave is incident on a piezoceramic cylinder, a nonuniform field of mechanical stresses is set up in it, and only within elementary layers of the cylinder, in three mutually perpendicular planes, are the mechanical stresses uniform. To eliminate the nonuniqueness in determining the mechanical stresses in a piezoceramic material, one must set up the boundary conditions for the forces acting on the faces of the piezoelectric element or for the mechanical energy entering the piezoelectric element through the corresponding face.

The following forces act on the acoustic boundaries of the piezoelectric element:

— in a longitudinal direction from the side of the acoustic medium \( F_z^1 = -Z_p \theta_z^1 \), and when there is no mechanical load on the upper end \( F_z^2 = 0 \);

— in a transverse direction, for the free cylindrical surfaces \( F_r^1 = 0 \) and \( F_r^2 = 0 \).

To determine the forces acting in a circular direction, we can use as the boundary condition the property of periodicity

\[ F_z(\theta) = F_z(\theta + 2\pi) . \]

The electrical boundary condition when the electrodes are open-circuited can be written in the following form:

\[ \int_{S_z} D_z dS = 0 . \]

The electrical induction \( D_z \) is related to the current \( I \) flowing through the piezoelectric element, the area of the end surface of which is \( S_z \), by the relation

\[ D_z = \frac{I}{J \omega S_z} . \]

Hence, for a piezoceramic cylinder we can write a system of boundary equations connecting the values of the forces acting on the faces of the piezoelectric element with the rate of displacement of the particles \( \theta_z^1 \) on the acoustic-medium-trans-