EFFECTS OF GAUSSIAN BEAM SIZE ON DIFFRACTION MEASUREMENT ERRORS

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A numerical simulation has been performed for the diffraction of a gaussian beam at a wire in the Fraunhofer approximation. The wire is symmetrically disposed with respect to the beam axis in the calculations on the shifts in the diffraction minima, while an unsymmetrical position has been used in calculating the diffraction pattern, which is characterized by change in the minima and maxima. Conditions are evaluated under which one can use the homogeneous-beam approximation in diffraction measurements.

Many optical measurement methods are based on Fraunhofer diffraction, particularly the measurement of particle size distributions by small-angle scattering [1] and a method of measuring the diameters of wires and optical fibers [2]. Standard instruments are produced for these measurements. They employ the diffraction of a planar homogeneous wave at various obstacles, e.g., a spherical particle or a filament. Various algorithms are used to determine the size: from the positions of the minima or maxima or else from the entire diffraction pattern. It is necessary to examine the limits to the utility of Fraunhofer diffraction in order to increase the accuracy in such measurements. For example, it has been shown [3] that there is a difference between Fraunhofer diffraction and the diffraction derived from the Lorentz-Mie theory. Fraunhofer diffraction is applicable in measuring the diameters of wires from 50 to 650 μm with errors of not more than 1%. Lasers are used as the sources, and there may be systematic errors in measuring particle size or wire diameter. The diffraction of a gaussian beam is different from that of a plane wave [4] and this can lead to new effects in laser Doppler anemometry, where it has been observed that the Doppler signal spectrum from a single large particle is split in a differential system [5].

We have used numerical simulation for various positions of the wire relative to the laser beam to examine how the diffraction from a gaussian beam affects the error in diffraction measurements.

We consider the diffraction of a gaussian beam at the narrowest point at a rectangular obstacle (Fig. 1). The laser beam 1 is directed to the obstacle 2, at which it is diffracted. The diffraction pattern is observed at the focus of the lens 3 in the recording plane 4. The width 2a of the obstacle is equal to the wire diameter, while the height 2b is much greater than the radius of the gaussian beam.

The field intensity distribution in a gaussian beam at the narrowest point is [1]

\[ E(\xi, \eta) = A_0 \exp \left( \frac{\xi^2 + \eta^2}{w_0^2} \right), \]

in which \( A_0 \) is the field strength at the beam axis and \( w_0 \) is the radius in the beam at which the field strength is reduced by a factor \( e \).

The power density distribution in the beam \( F_1 \) is dependent on the beam power \( P_0 \) and on the radius:

\[ F_1(\xi, \eta) = \frac{2P_0}{\pi w_0^2} \exp \left( -2 \frac{\xi^2 + \eta^2}{w_0^2} \right) \]

We calculate the Fraunhofer diffraction at a planar screen by neglecting the fact that the surface of the wire is cylindrical. The error is then slight [3]. Babinet’s principle shows that the diffraction at a planar screen may be replaced by
Diffraction at a slit, with differences observed only in the central region, whose dimensions are determined by the beam parameters. It has been shown [6] to be permissible to replace a cylinder by a planar screen. If the origin is at the center of the rectangle having dimensions $2a \times 2b$, while the $O\xi$ and $O\eta$ axes are parallel to the sides, then the expression for Fraunhofer diffraction at an obstacle of any shape for a gaussian wave is [6]

$$E_D(x,y) = \frac{k A_0}{2\pi f} \exp\{jkf\} \int_{-a}^{a} \int_{-b}^{b} \exp\left\{-\frac{x^2 + y^2}{w_0^2}\right\} \exp \left\{ -jk\left(\frac{x^2}{f} + \frac{y^2}{f}\right)\right\} dx dy,$$

in which $x$ and $y$ are coordinates in the recording plane, $\xi$ and $\eta$ are coordinates in the plane containing the wire, $k$ the wave vector modulus, $k = 2\pi/\lambda$, $\lambda$ the wavelength, and $f$ the focal length of the lens.

The power density in the diffraction pattern is

$$F_D(x, y) = \frac{\rho_0}{\pi w_0^2 f^2} \left[ \int_{-a}^{a} \int_{-b}^{b} \exp\left\{-\frac{x^2 + y^2}{w_0^2}\right\} \exp \left\{ -jk\left(\frac{x^2}{f} + \frac{y^2}{f}\right)\right\} dx dy \right]^2.$$

As $b \gg W_0$, (1) becomes as follows for the plane $y = 0$;

$$F_D(x, 0) = \frac{\rho_0}{\pi^2 f^2} \left[ \int_{-a}^{a} \exp\left\{-\frac{x^2}{w_0^2}\right\} \cos \left(2\pi x \xi / \lambda f\right) dx \right]^2.$$

From (2) we get the directed power density in the recording plane for a wire at the center of the beam.

We simulated the diffraction of a gaussian beam from an He–Ne laser having $\lambda = 0.6328 \mu m$ for various beam radii and wires with various diameters.

Figure 2 shows $f(x) = F_D(x)/F_D(0)$ for $w/a = 1.5$ and 1.0, and also for comparison the power density distribution for a diffracted planar homogeneous wave ($w/a = \infty$), in which case

$$f(x) = \frac{F_D(x)}{F_D(0)} = \left[ \frac{\sin \left(2\pi ax / \lambda f\right)}{2\pi ax / \lambda f} \right]^2.$$

The graphs show that the gaussian beam radius affects the positions of the diffraction minima and maxima and the power densities in the maxima. As that radius increases, the pattern approaches that for a planar homogeneous wave. Table 1 gives the relative shifts in the diffraction maxima in relation to the relative beam size as calculated from $\bar{x} = (x_n - x_{no})/x_{no}$, in which $x_n$ and $x_{no}$ are the positions of minimum $n$ in diffraction respectively of a gaussian beam and a plane wave. The largest shift occurs for the first diffraction minimum. This means that if diffraction measurements are made to determine the