A TEST FOR INDEPENDENCE OF TWO STATIONARY INFINITE ORDER AUTOREGRESSIVE PROCESSES

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Abstract. This paper considers the independence test for two stationary infinite order autoregressive processes. For a test, we follow the empirical process method and construct the Cramér-von Mises type test statistics based on the least squares residuals. It is shown that the proposed test statistics behave asymptotically the same as those based on true errors. Simulation results are provided for illustration.

Key words and phrases: Independence test, infinite order autoregressive processes, the Cramér-von Mises test, residual empirical process, weak convergence.

1. Introduction

In this paper, we consider the problem of testing the independence of two stationary time series. For the past two decades, the issue has drawn much attention from many researchers. For instance, Haugh (1976) proposed an independence test for the errors in ARMA models based on the sum of squares of residual cross correlations. Later, adopting his idea, Pierce (1977), Geweke (1981) and Hong (1996) studied the independence test for two stationary time series. In fact, the method using the cross correlations has been much popular in the time series context since it is a crucial task to figure out the dependence structure of given time series in a correct manner, and any model selection procedures require a step for diagnostics to set up a true model. However, the cross correlation method merely guarantees the uncorrelatedness of observations and does not ensure the independence. Moreover, the cross correlation just checks the linear relationship but cannot find a nonlinear dependence. Therefore, instead of it, here we employ the empirical process method devised by Hoeffding (1948) and Blum et al. (1961). Their test statistics essentially fall into the category of Cramér-von Mises (CV) statistics, and have been applied under a variety of circumstances. Recently, their method has been adopted by Skaug and Tjøstheim (1993), Delgado (1996), Hong (1998) and Delgado and Mora (2000) aimed at developing a serial independence test.

In this paper, we focus on the independence test for two stationary infinite order autoregressive processes. We adopted autoregressive processes since they include the most popular ARMA processes in time series analysis, and a method based on residuals usually discards correlation effects. In fact, the infinite order autoregressive process has been in a central position in the study of the asymptotic efficiency of model selection criteria (see Shibata (1980), Lee and Karagrigoriou (2001) and the papers therein). For a test, we construct the CV statistic based on residuals. It will be seen that the limiting distribution of the residual based CV statistic is the same as the CV statistic based on the true errors.
In fact, the CV statistic is designed for testing the independence for a specific lag \( k \). However, in real situations one should check the independence for several lags, say, \( |k| \leq K \), where \( K \) is a positive integer larger than 1, since the test for only one lag is not sufficient to ensure the independence between the two times series. To this end, we consider three types of test statistics. First, we consider the summation of the CV statistics based on residuals, which Skaug and Tjostheim (1993) suggested as a test statistic for testing serial independence. Second, we consider the weighted summation of the CV statistics which was proposed in Hong (1998) for testing serial independence, since the summation type test statistic may suffer from severe size distortions as \( K \) increases. Finally, we propose as a test statistic the maximum of the CV test statistics. We consider this because it is less affected by the CV statistics with large values and will have more stability compared to other tests. Our simulation study shows that the method based on the CV statistic with residuals turns out to be suitable for the independence test of two stationary time series.

The rest of the paper is organized as follows. In Section 2, we present the procedure for the independence test of two stationary infinite order autoregressive processes. In particular, we derive the asymptotic distribution of the proposed test statistics. This task requires extending the result of Lee and Wei (1999) to the residual empirical process with bivariate time parameters, which may be of independent interest in its own sake. In Section 3, we report the result of our simulation study. Finally, in Section 4 we provide the proofs of the theorems presented in Section 2.

2. Main results

Suppose that \( \{X_t\} \) and \( \{Y_t\} \) satisfy the following difference equations:

\[
X_t - \mu - \sum_{j=1}^{\infty} \phi_j (X_{t-j} - \mu) = \epsilon_t, \quad t = 1, \ldots, n
\]

and

\[
Y_t - \nu - \sum_{j=1}^{\infty} \theta_j (Y_{t-j} - \nu) = \eta_t, \quad t = 1, \ldots, n,
\]

where \((\epsilon_t, \eta_t)\) are random vectors with common distribution \( F \), \( \epsilon_t \) and \( \eta_t \) are iid r.v.'s with marginal distributions \( F_1 \) and \( F_2 \), respectively, and \( E\epsilon_t^4 + E\eta_t^4 < \infty \). Furthermore, both \( A_1(z) := 1 - \sum_{j=1}^{\infty} \phi_j z^j \) and \( A_2(z) := 1 - \sum_{j=1}^{\infty} \theta_j z^j \) are assumed to be analytic on an open neighborhood of the closed unit disk \( D \) in the complex plane and have no zeroes on \( D \). It can be easily seen that the last condition implies

\[
|\phi_j| + |\theta_j| \leq Cp^j, \quad C > 0, \quad 0 < \rho < 1,
\]

(cf. Lee and Wei (1999)). It is well-known that the AR(\( \infty \)) process covers a broad class of stationary processes including invertible ARMA processes (cf. Brockwell and Davis (1990)).

Suppose that one wishes to test the hypotheses

\[ H_0 : \{X_t\} \text{ and } \{Y_t\} \text{ are independent.} \quad \text{vs.} \quad H_1 : \text{not } H_0. \]

The above is equivalent to testing

\[ H'_0 : \{\epsilon_t\} \text{ and } \{\eta_t\} \text{ are independent.} \quad \text{vs.} \quad H'_1 : \text{not } H'_0. \]