DOUBLE-EXPOSURE RECORDING OF A LENSLESS FOURIER SPECTROGRAM FOR FORMING A SHEAR INTERFEROGRAM

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Results of theoretical estimation of the sensitivity of a shear speckle interferometer intended for testing of wavefront aberrations are presented. It is shown that in the case of the spherical aberration the sensitivity of the interferometer is doubled to an accuracy of 12.3% for a fixed shear magnitude.

In [1], it was shown that in the case of double-exposure recording of a lensless Fourier spectrogram, when prior to the second exposure of a photoplate a mat screen had been shifted in its plane, a shear interferogram in fringes of infinite width was formed in the stage of reproduction of a record, which characterized the spherical wavefront aberration of coherent radiation used to illuminate the mat screen. In so doing, the interference pattern is localized in the far diffraction zone, and the differential interferometer of this type has a better sensitivity for a fixed shear magnitude.

In this paper, the results of theoretical estimation of the sensitivity of such a speckle interferometer are presented.

Unlike [1], in the general case where the curvature radius R of a converging quasi-spherical wave in the \((x_1, y_1)\) plane of the mat screen does not coincide with the distance \(l\) from the screen to the photoplate, in the Fresnel approximation the distribution of the complex field amplitude in the \((x_2, y_2)\) photoplate plane corresponding to the first exposure assumes the form

\[
U_1(x_2, y_2) = \exp\left[i\frac{k(x_2^2 + y_2^2)}{2l}\right] \{F(x_2, y_2) \ast \Phi(x_2, y_2) \ast \exp[-i k(x_1, y_1)]\},
\]

where constant factors are neglected, \(\ast\) denotes the convolution operation, \(k\) is the wave number,

\[
F(x_2, y_2) = \int \int t(x_1, y_1) \exp[-i k(x_1, x_2 + y_1, y_2)] dx_1 dy_1
\]

is the Fourier transform of the random function \(t(x, y)\) of coordinates describing the complex transmission amplitude of the mat screen,

\[
\Phi(x_2, y_2) = \int \int \exp[i \varphi(x_1, y_1)] \exp[-i k(x_1, x_2 + y_1, y_2)] dx_1 dy_1
\]

is the Fourier transform of a complex function, and \(\varphi(x_1, y_1)\) is the deterministic function describing phase distortions in the wavefront of coherent radiation used to illuminate the mat screen produced by aberrations in the optical system forming it.

If before the second exposure the mat screen is displaced in its plane by a distance \(a\), for example, in the direction of the \(x\) axis, the distribution of the complex field amplitude in the photoplate plane, corresponding to the second exposure, will be determined by the expression

\[
U_2(x_2, y_2) = \exp[i k(x_2^2 + y_2^2)/2l] \{\exp[i k \alpha x_2/l] F(x_2, y_2) \ast \Phi(x_2, y_2) \ast \exp[-i k(x_1, y_1)]\},
\]

Following [2] and taking \(\exp[i k \alpha x_2/l]\) outside the convolution integral, we will reduce Eq. (2) to the form
\[
U_2(x_2, y_2) \sim \exp\left[i\frac{(kx_2^2 + y_2^2)}{2l}\right]\{\exp(ikax_2/l)F(x_2, y_2) \otimes \exp(-ikx_2/l)\}.
\]

When the doubly exposed objective speckle structures are recorded in the linear section of the characteristic curve for blackening of a photographic material, the transmission of the specklogram will be proportional to (without regard for the constant component)

\[
U_1(x_2, y_2)U_1^*(x_2, y_2) + U_2(x_2, y_2)U_2^*(x_2, y_2).
\]

In analogy with [1], below we assume that in the reproduction stage the double-exposed specklogram is illuminated with a plane wave from a coherent light source used in the recording stage. Then in the \((x_3, y_3)\) back focal plane of the positive lens realizing the Fourier transform of the field scattered by the specklogram the distribution of its complex amplitude assumes the form

\[
U(x_3, y_3) \sim \left\{ i(-x_3, -y_3)\exp\left[-i\frac{(kx_3^2 + y_3^2)}{2Rl}\right]\otimes \right. \\
\times \exp\left[-i\varphi(x_3, y_3)\right]\exp\left[i\frac{(kx_3^2 + y_3^2)}{2Rl}\right] + \exp\left[i\frac{ikax_3(R-1)}{Rl}\right]\{i(-x_3, -y_3)\} \\
\times \exp\left[-i\frac{(kx_3^2 + y_3^2)}{2Rl}\right] + \exp\left[i\frac{ikax_3(R-1)}{Rl}\right] \otimes t^*(x_3, y_3) \exp(-i\varphi(x_3 - a, y_3)) \\
\times \exp\left[-i\frac{(kx_3^2 + y_3^2)}{2Rl}\right] + \exp\left[i\frac{ikax_3(R-1)}{Rl}\right] \otimes P(x_3, y_3),
\]

where \(P(x_3, y_3)\) is the Fourier transform of the generalized pupil function of the lens [1]; in addition, for brevity we assume that the focal length of the lens realizing the Fourier transform of the field is equal to \(l\).

From relation (4) it follows that in the Fourier plane the superposition of two identical subjective speckle fields is formed with the speckle size determined by the width of the function \(P(x_3, y_3)\). In this case, the speckle fields of two exposures are tilted relative to each other at an angle determined by the phase factor \(\exp[ikax_3(R-1)/Rl]\).

Let us write down an expression for the light-intensity distribution in the \((x_3, y_3)\) plane. To exclude from consideration the speckle-modulation of the light field, we perform averaging over the coordinates. To this end, we assume that the diameter of the area over which the averaging is performed is much greater than the subjective speckle size. At the same time, the phase factor \(\exp[ikax_3(R-1)/Rl]\) responsible for the formation of the equidistant interference fringes remains unchanged within this area. Then for the average intensity \(I(x_3, y_3)\) in the Fourier plane, we will obtain

\[
I(x_3, y_3) = \langle A \rangle + \langle B \rangle + 2\Re\langle C \rangle \cos[kax_3(R-1)/Rl] - 2\Im\langle C^* \rangle \sin[kax_3(R-1)/Rl],
\]

where \(\Re\) and \(\Im\) denote real and imaginary parts, respectively; the angular brackets denote averaging;

\[
\langle A \rangle = \iiint\int\langle D \rangle \exp\{i\varphi(-\zeta, -\eta) - \varphi(-\zeta_1, -\eta_1) - \varphi(x_3 - \zeta, y_3 - \eta) + \varphi(x_3 - \zeta_1, y_3 - \eta_1)\}\exp i\beta d\zeta d\eta d\zeta_1 d\eta_1,
\]

\[
\langle B \rangle = \iiint\int\langle D \rangle \exp\{i\varphi(-\zeta - a, -\eta) - \varphi(-\zeta_1 - a, -\eta_1) - \varphi(x_3 - \zeta - a, y_3 - \eta) + \varphi(x_3 - \zeta_1 - a, y_3 - \eta_1)\}\exp i\beta d\zeta d\eta d\zeta_1 d\eta_1,
\]

\[
\langle C \rangle = \iiint\int\langle D \rangle \exp\{i\varphi(-\zeta, -\eta) - \varphi(-\zeta_1, -\eta_1) - \varphi(x_3 - \zeta, y_3 - \eta) + \varphi(x_3 - \zeta_1, y_3 - \eta_1)\}\exp i\beta d\zeta d\eta d\zeta_1 d\eta_1,
\]

\[
\langle D \rangle = \langle \epsilon(-\zeta, -\eta) \rangle \langle x_3 - \zeta, y_3 - \eta \rangle \langle \epsilon(-\zeta_1, -\eta_1) \rangle (R/l),
\]

\[
\beta = k[x_3(\zeta - \zeta_1) + y_3(\eta - \eta_1)] (R/l).
\]