METHOD OF ORIENTED PSEUDOCONVECTION

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In the present paper a new method is suggested for representing convective terms in a multidimensional differential transfer equation by a numerical scheme of increased order of accuracy with stability close to that of the well-known difference scheme for the counterflow, which at the same time does not introduce the schematic diffusion in the calculations. Among the advantages of the method are its simplicity and high efficiency of numerical calculations of the multidimensional transfer equations for large Reynolds and Peclet numbers.

INTRODUCTION

An important problem in the construction of algorithms for numerical solution of the transfer equation is the creation of a stable scheme for the approximation of the convective terms by difference expressions of the second order of accuracy. Many approximation schemes of this type are unstable for high flow rates of a fluid, and the most stable schemes with differences counter the flow have insufficient accuracy for investigation of complex 3-D flows of a viscous fluid. Schemes with central differences, differences counter the flow, hybrid, power-law, and many others either lose their stability for large Reynolds numbers, have insufficient accuracy, or are cumbersome and expensive for numerical solution of multidimensional nonstationary problems of fluid dynamics. In the overwhelming majority of cases, difference analogs of increased order of accuracy are written for four or five points of the difference scheme along any coordinate, which substantially complicates the numerical calculations, and though these schemes have low schematic diffusion, most of them introduce oscillations and frequently result in loss of convergence.

1. SUGGESTED METHOD

In the present paper the approach [1] is suggested whose essence is reduced to the following. The convective terms of the equation of transfer of a certain substance are written on the basis of the well-known difference scheme having the required accuracy (for example, on the basis of the three-point scheme with central differences of the second order of accuracy). It is well known that when the Reynolds grid numbers are greater than two, this scheme is unstable; therefore, an additional stabilizing term is introduced in the transfer equation (1b), which allows a stable solution of the problem to be obtained. The introduced correction ensures a stable scheme with differences counter the flow. At the same time, it tends to zero in the process of numerical solution. After the iterations have converged, the stabilizing term vanishes, and the approximation of the convective terms will have the second order of accuracy according to the scheme with central differences. The advantages of the present approach are the absence of schematic viscosity that appears in other schemes of approximation of the convective terms for a Reynolds grid number greater than two and the feasibility of combining high stability of the scheme with differences counter the flow and the second order of accuracy of the scheme with central differences.

To illustrate the method, we consider a solution to the 3-D problem on the spread of a scalar substance. The transfer equation in this case usually has convective, diffusive, and source terms and is said to be the 3-D elliptic differential equation. The accuracy of numerical solution of the transfer equation for high velocities (the case where the Reynolds grid number is greater than two is meant) depends mainly on the accuracy of convection representation, because the use of the well-known difference analogs of the second order of accuracy for complex 3-D flows tends to be unstable. Therefore, for brevity of presentation of the suggested method, below we consider the difference analogs only for the convective transfer of a substance.

Let us consider the simplest case of representation of the convective terms by the scheme with central differences of the...
second order of accuracy. In this case, stable convergence is obtained by introducing in the transfer equation, the oriented stabilizing pseudoaddition of the form

$$\frac{\partial \theta}{\partial t} + U_x \frac{\partial \theta}{\partial x} + U_y \frac{\partial \theta}{\partial y} + U_z \frac{\partial \theta}{\partial z} +$$

$$+ \left( \frac{\text{abs}(U_x)}{\Delta x} + \frac{\text{abs}(U_y)}{\Delta y} + \frac{\text{abs}(U_z)}{\Delta z} \right) \left( \theta_{i+1,j,k}^n - \theta_{i,j,k}^n \right) = \Phi ,$$

where $\theta$ denotes the transferable scalar substance; $U_i$ are the components of the velocity vector; $t$ is time; $\Delta x, \Delta y, \Delta z$ are the steps of the difference grid along the coordinate axes; $\Phi$ denotes the function containing diffusion, sources, sinks, etc.; subscripts $i, j, k,$ and $n$ denote the numbers of the difference grid along the coordinate axes and time (or the iteration number for the stable problem), respectively; $\text{abs}(U)$ denotes the modulus of $U$.

Let us consider, for definiteness, the stable problem of the transfer of a certain substance. The so-called method of temporal stabilization is used to solve this problem. In this case the subscript $n$ in Eq. (1b) characterizes the definite step of the process of temporal stabilization of the solution.

From an analysis of Eq. (1) we conclude that, after stabilization of the solution, i.e., when the sought-after quantities $\theta_{i+1}^n$ and $\theta_i^n$ on the left side of Eq. (1b) do not differ, the additional stabilizing term specified by Eq. (1b) will vanish. This means that the obtained solution will have an order of accuracy corresponding to that of the convective terms (here we discuss only the convective transfer and need not dwell on the right side of the equation). Thus, this way of introducing the additional stabilizing term, on the one hand, does not introduce any numerical diffusion, and on the other hand, as numerical studies have shown, ensures stable convergence.

Theoretical treatment of stability of the suggested method by the well-known methods [2] (for example, by the method of discrete perturbations, the Neumann method, or the Hert method) is inefficient because of its complexity; therefore, we restrict ourselves to an approximate analysis. To demonstrate the stability of the given method, we first consider the convective term with the stabilizing pseudoaddition along the $x$ axis (the subscripts $j$ and $k$ will be omitted). Convective transfer is represented in the discrete form with the use of the scheme with central differences. As a result, we obtain

$$U_x \frac{\partial \theta}{\partial x} + \frac{\text{abs}(U_x)}{\Delta x} \left( \theta_{i+1}^n - \theta_i^n \right) = U_x \frac{\theta_{i+1}^{n+1} - \theta_{i+1}^n}{2\Delta x} - \frac{\text{abs}(U_x)}{\Delta x} \theta_i^n + \frac{\text{abs}(U_x)}{\Delta x} \theta_{i+1}^{n+1} .$$

If the relation

$$\theta_i^n = \frac{1}{2} (\theta_{i+1}^n + \theta_{i-1}^n)$$

is satisfied, the right side of Eq. (2) can be reduced to the form

$$[U_x - \text{abs}(U_x)] \frac{\theta_i^{n+1} - \theta_{i-1}^n}{2\Delta x} - [U_x + \text{abs}(U_x)] \frac{\theta_i^{n+1} + \text{abs}(U_x)}{2\Delta x}.$$

Depending on the sign of the velocity $U_x$, Eq. (4) assumes the forms

$$U_x > 0 , \quad \text{abs}(U_x) \frac{\theta_i^{n+1} - \theta_{i-1}^n}{\Delta x} - U_x \frac{\theta_i^0}{\Delta x} ;$$

$$U_x < 0 , \quad \text{abs}(U_x) \frac{\theta_i^{n+1} + \theta_{i+1}^n}{\Delta x} + U_x \frac{\theta_i^0}{\Delta x} .$$

The dependences (5) are nothing but the representation of the convective transfer by a scheme with differences counter the flow [2], which, as is well known, is stable.