LENS EFFECT IN THE WAVEGUIDE GAS LASER

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Abstract

The radiation field of a planar waveguide laser with a nonuniformly heated gas causing the lens effect was studied. Conditions were found at which the losses at the walls and nonlinearity of the active medium (saturation) became essential. Calculations of the lasing of a xenon laser were performed with regard for the concurrence of modes. The radiation field distributions in the near and far diffraction zones were calculated. The results of calculations correlate qualitatively with experimental data.

1. Introduction

Virtually all of the studies on waveguide gas lasers (see, e.g., [1–3]) were based on the concept of the structure of the radiation field in an active medium as the empty waveguide modes, considered in [4]. However, it was recently found [5] that the thermal lens effect appreciably influenced the structure of the radiation field in the xenon laser at gas pressures of the order of 100 torr. In the first papers devoted to the studies of this operation mode [5, 6], such factors as radiation losses at the waveguide walls and the saturation effect were not taken into account. In the present paper, these phenomena are considered and an attempt is made to find the distribution of the lasing field using the simplified model of the concurrence of modes.

2. Waveguide-Mode Losses

Let us consider the radiation propagation in a planar waveguide. We suppose that the radiation field depends on two Cartesian coordinates — x (transverse coordinate) and z (coordinate in the direction of radiation propagation). Let us assume that the waveguide walls are at x = ±l (the waveguide thickness is 2l).

We do not consider saturation initially. Then, the dependence of the field on the z coordinate for the particular waveguide mode is reduced to the \( \exp(ikz) \) factor, where k is a constant that is not equal in the general case to the wave number of radiation in vacuum \( k_0 \). The field equations can be represented in the form (cf. [7])

\[
\frac{d^2 f}{dx^2} + \left[ \varepsilon(x)k_0^2 - k^2 \right] f = 0,
\]

where \( f = E_y \) and \( f = D_z \) for TE and TM waves, respectively. Strictly speaking, the equation for \( D_z \) is somewhat different from (1), but for gas having permittivity \( \varepsilon(x) = 1 + \delta \varepsilon(x) \) and \( \delta \varepsilon(x) \ll 1 \) this difference is insignificant.

The boundary conditions at the waveguide walls have to be added to Eq. (1) to determine the field in the waveguide. Since the fields under consideration do not depend on y, it is obvious that only plane (inhomogeneous) waves propagate in waveguide walls. These waves possess \( k' \) wave vector having only x and...
\( z \) components. As follows from the continuity of the tangent components of electric and magnetic fields at
the waveguide boundary, \( k'_z = k \). The \( k'_z \) component can be found from the Maxwell equations for the field
in the waveguide plate.

For definiteness, consider the TM wave. One can derive for the fields at the surface of plates from the
Maxwell equations in the waveguide space

\[
H_y(l) = \frac{k_0}{k} D_x(l); \quad E_z(l) = \frac{i}{k \varepsilon(l)} \frac{dD_x(l)}{dx}.
\]

Similar values of these components of the field should be boundary parameters for the wave inside the
waveguide due to the corresponding continuity conditions. The continuity of the normal component of the
\( D \) vector serves as the third condition.

Given the said boundary conditions, one can easily obtain from the Maxwell equations for the field in the
waveguide plate

\[
k'_z = -i \frac{\varepsilon_0}{\varepsilon(l)} \frac{1}{D_x(l)} \frac{dD_x(l)}{dx},
\]

where \( \varepsilon_0 \) is the permittivity of the waveguide-wall material. Therefore, the following condition must be
satisfied:

\[
\varepsilon_0 k^2 = (k'_z)^2 + (k'_z)^2 = k^2 - \frac{\varepsilon_0^2}{\varepsilon^2(l)} \left[ \frac{1}{D_x(l)} \frac{dD_x(l)}{dx} \right]^2.
\]

It is convenient to represent Eq. (1) in the form

\[
\frac{d^2 f}{dx^2} + [\nu(x) + q] f = 0,
\]

where \( \nu(x) = \delta \varepsilon(x) k^2 \) and the constant \( q = k^2 - k'^2 \). Equation (2) can be rewritten now in the form

\[
k^2 (\varepsilon_0 - 1) = q - \frac{\varepsilon_0^2}{\varepsilon^2(l)} \left[ \frac{1}{D_x(l)} \frac{dD_x(l)}{dx} \right]^2.
\]

At \( k_0l \gg 1 \), the inequality \( |q| \ll k^2 (\varepsilon_0 - 1) \) is accomplished easily. Thus the boundary condition

\[
f(q, l) = 0
\]

is a good approximation and is usually employed for the determination of waveguide modes (see, e.g., [5]).

The second boundary condition (at \( x = -l \)) has the same form, but in the symmetric waveguide the
modes of linear approximation must be even or odd functions of \( x \). Therefore, for a symmetric waveguide it
is sufficient to perform the calculation in the interval \( 0 < x < l \) and call for \( f(q, 0) = 0 \) and \( \nabla_x f(q, 0) = 0 \),
respectively, for odd and even modes as the second boundary condition.

The correction to the \( q \) value along with the loss value can be found from Eq. (4), expanding \( f(q, l) \) into
a series of \( \Delta q = q - q_0 \) (\( q_0 \) is the solution of Eq. (5)) and retaining only the first term of the series:

\[
\Delta q = \pm i \frac{\varepsilon_0}{k_0 \sqrt{\varepsilon_0 - 1}} \frac{\nabla_x D_x(q_0, l)}{\nabla_x D_x(q_0, l)}
\]

(the TM mode is implied here). One can find the correction to the wave number \( k \) now, since \( k^2 = k_0^2 - q \)
(see above):

\[
k = \sqrt{k_0^2 - q} \approx k_0 - \frac{q_0 + \Delta q}{2k_0}.
\]