GRAVITATIONAL SCALAR-FIELD INTERACTION IN AFFINE METRIC GRAVITATIONAL THEORY

V. G. Krechet and D. V. Sadovnikov

Gravitational scalar-field interaction is examined on the basis of possible effects from torsion and nonmetrical space-time within the framework of affine-metric gravitational theory. Cosmological and astrophysical topics are examined as applications of the theory.

Various extensions of Einstein's gravitational theory remain of interest to research on eliminating difficulties in GRT and in geometrizing fundamental physical interactions. On the gauge approach to gravitational theory, one uses the symmetry groups of Minkowski space: the Lorentz-Poincaré group and also the dilatation group $D: x^a \rightarrow \lambda x^a, x_a \rightarrow (1/\lambda)x_a$, in which $\lambda = $ constant, which on localization as gauge fields result in various types of connectedness in space-time.

When the Lorentz group is localized as a gauge field, we get Riemann-Cartan connectivity: unsymmetrical connectedness with torsion. If we further incorporate another symmetry type for Minkowski space (the dilatation group), localization of that group gives us the Weil nonmetrical vector $W_t$ as the gauge field [1]:

$$\nabla_i g_{kl} = 2 W_i g_{kl}. \quad (1)$$

We thus get a space containing torsion and Weil-type nonmetricity. Therefore, in this theory we envisage a space in which the nonmetricity and torsion are not zero.

The connectedness coefficients are as follows in the general case in an affine-metric space of that type:

$$\Gamma^i_{jk} = \Gamma^i_{[jk]} + \frac{1}{3} (\delta^i_k Q^l + \delta^i_j Q^l - \delta^i_l Q^j - \delta^i_l Q^j + \delta^i_l Q^j) + \varepsilon^{lmn}_{[jk]} \dot{Q}^m. \quad (2)$$

Here $Q^l_{[jk]} = \Gamma^i_{[jk]}$ is the torsion tensor, which is represented in terms of its irreducible components [2] as follows:

$$Q^l_{[jk]} = -\frac{1}{2} Q^l_{[jk]} + \frac{1}{3} (\delta^l_k Q^i - \delta^l_j Q^i) + \varepsilon^{lmn}_{[jk]} \dot{Q}^m. \quad (3)$$

in which $\dot{Q}^m = (1/6)\varepsilon^{mn} Q_{nm}$, $\dot{Q}^m$ is the pseudotrace of the torsion tensor, $Q^i = Q^i_{ll}$ is the trace of the torsion tensor, and $Q^l_{[jk]}$ is the trace-free part of the torsion tensor. In this case, the curvature scalar $\tilde{R}$ is represented as

$$\tilde{R} = R - 6 W_i Q^i + 4 Q^i Q_i - 8 W_i W_i. \quad (4)$$

Here we do not consider the trace-free part of the torsion tensor because the sources of it are absent in this particular case.

We take the gravitational lagrangian in accordance with GRT as the curvature scalar of this affine-metric space, which is the immediate covariant extension of the GRT gravitational lagrangian:

$$L_g = \frac{-\tilde{R}}{2x}. \quad (5)$$
The material source of the gravitational field is taken as a scalar field having a conformal connection described by the lagrangian $L_m$:

$$L_m = -\frac{1}{2}\left(\varphi_{,t} \varphi^t - \frac{\tilde{R}}{6} \varphi^2 + \mu^2 \varphi^2\right).$$

After one has varied the action having the lagrangian $L = L_g + L_m$ with respect to the geometrical quantities $g_{ik}$, $W^t$, $Q^t$, $\chi^t$ and the scalar field $\varphi$, one gets an equation system for the gravitational fields and matter:

$$16 \left(\varphi_{,t} \varphi^t - \frac{\tilde{R}}{6} \varphi^2 + \frac{\mu^2}{2} \varphi^2\right) +
$$

$$+ \frac{8}{3} \left(Q^t Q_{\kappa} - \frac{1}{2} Q^t Q_{t} g_{ik}\right) + 8 \left(W^t Q_{\kappa} - \frac{1}{2} W^t Q_{t} g_{ik}\right) -
$$

$$- 6 \left(Q_{t}^t Q_{\kappa} - \frac{1}{2} Q_{t} Q_{\kappa} g_{ik}\right) \left(1 - \frac{x}{6} \varphi^2\right) + \frac{x}{2} \mu^2 \varphi^2 g_{ik} -
$$

$$- \frac{x}{6} \left[6 \left(\varphi_{,t} W_{\kappa} - \frac{1}{2} \varphi_{,t}^t W^t g_{ik}\right) + 4 \left(\varphi_{,t} Q_{\kappa} - \frac{1}{2} \varphi_{,t} Q_{t} g_{ik}\right) -
$$

$$- \frac{x}{6} \left((\varphi_{,t} \varphi_{,\kappa} - \varphi^t \varphi_{,t} g_{ik}) \varphi^2 + \varphi_{,t} \varphi_{,\kappa} - \frac{1}{2} \varphi^t \varphi_{,t} g_{ik}\right)\right];
$$

$$a) \quad G_{ik} = 6 \left(V_{t, t}^t V_{\kappa} - \frac{1}{2} V^t V_{t} g_{ik}\right) + \frac{3x}{1 - \Phi^2} \mu^2 \Phi^t g_{ik} -
$$

$$- \frac{6}{1 - \Phi^2} \left[\Phi_{t, t}^t V_{\kappa} - \frac{1}{2} \Phi_{t}^t \Phi_{t} V_{t} g_{ik} + \frac{1}{6} (\varphi_{,t} \varphi_{,\kappa} - \varphi^t \varphi_{,t} g_{ik}) \Phi^t +
$$

$$+ \Phi_{,t} \Phi_{,\kappa} - \frac{1}{2} \Phi^t \Phi_{,t} g_{ik}\right];
$$

$$b) \quad V_{t} = \frac{\Phi_{t} \Phi_{,t}^t}{1 - \Phi^2};
$$

$$c) \quad \Phi_{,t} + \frac{1}{6} \Phi^t = \mu^2 \Phi.$$

It follows from (7c) that the torsion pseudotrace $\chi^t = 0$ may interact with the scalar field only via its trace. It is also readily seen that the latter two equations in (7) for the torsion trace and Weil vector coincide, i.e., the nonmetricity and the torsion in this theory are described by a single linear algebraic equation.

If we introduce the resultant vector

$$3V_{t} = 2Q_{t} + 3W_{t},$$

then the dynamics of the scalar field in this affine-metric space will be determined by the following self-consistent equation system (here and subsequently $\Phi = \varphi \sqrt{\kappa/6}$):

$$\begin{align*}
\text{a) } & \quad G_{ik} = 6 \left(V_{t, t}^t V_{\kappa} - \frac{1}{2} V^t V_{t} g_{ik}\right) + \frac{3x}{1 - \Phi^2} \mu^2 \Phi^t g_{ik} - \\
& - \frac{6}{1 - \Phi^2} \left[\Phi_{t, t}^t V_{\kappa} - \frac{1}{2} \Phi_{t}^t \Phi_{t} V_{t} g_{ik} + \frac{1}{6} (\varphi_{,t} \varphi_{,\kappa} - \varphi^t \varphi_{,t} g_{ik}) \Phi^t + \\
& + \Phi_{,t} \Phi_{,\kappa} - \frac{1}{2} \Phi^t \Phi_{,t} g_{ik}\right]; \\
\text{b) } & \quad V_{t} = \frac{\Phi_{t} \Phi_{,t}^t}{1 - \Phi^2}; \\
\text{c) } & \quad \Phi_{,t} + \frac{1}{6} \Phi^t = \mu^2 \Phi.
\end{align*}$$

Here $\tilde{R} = R\{\} + 6V_{t, t} - 6VV_{t}$, and the only non-Riemann characteristic is provided by the resultant vector (8).

The first term in the effective energy-momentum tensor in (9a) coincides with the energy-momentum tensor for an ideal liquid having the equation of state $p = \Xi$, where the vector $V_{t}$ is proportional to the 4-velocity vector for the effective ideal liquid: $V_{t} = U \sqrt{\kappa p + \Xi}$. that is, the effects from the non-Riemann objects that the scalar field may interact with are equivalent to the presence of a certain ideal liquid.