BASIC RELATIONS OF THE THEORY OF NONLINEAR DYNAMIC DEFORMATION
OF HIGH-STRENGTH POLYMER FILMS

Ya. F. Kayuk and A. A. Sikora

We present basic relations of the theory of nonlinear dynamic deformation of rigid (high-strength) polymeric films. On the basis of the experimental results, we formulate the determining relations for the elastic and plastic stages of deformation. We introduce a measure of the stress-strain state and obtain the equations of motion for arbitrary curvilinear orthogonal coordinates in the case of large displacements and low strains and squares of the angles of rotation.

The polymeric films extensively used in contemporary engineering can be split into two classes: elastic (elastomers) and rigid. Theoretical foundations of the deformation of elastomers were developed, e.g., in [1–7]. These materials can be deformed by about 800–900% and, in the general case, the relations between the forces and strains are physically and geometrically nonlinear.

In recent years, we observe the appearance of new types of films capable of withstanding significant tensile forces and transverse shock loads. For these films, fracture strains are 10–20 times lower than for elastomers. Films of this sort are used as antishock systems in astronautics, industry, etc.

Earlier, to check the dynamic strength of the films, we carried out approximate experiments* in which restrained one-layer and two-layer square films with a side of 0.54 m were subjected to impacts of a metallic ball 4.11 kg in mass falling from a height of 6 m. A hole appeared in the two-layer films after 12–14 impacts. Therefore, we found it necessary to undertake the experimental investigation of the deformability and mechanical properties of these films with the aim of developing a theory of their nonlinear dynamic deformation.

Since, in practice, rigid films are usually delivered in rolls, for the analysis of their mechanical properties and the degree of anisotropy, we cut out specimens with working area 20 x 5 mm in size along and across 80-mm-long rolls. The width of the grips (in a testing machine) was equal to 15 mm. For testing, we used films with a thickness of 3.6 μm produced by the ARMA SHIELD firm (United Kingdom).

We carried out experimental investigations in a TIRATEST-2151 universal machine in the Laboratory of mechanics of polymeric and composite materials of the Kiev National Technical University. The results obtained for specimens subjected to tension at a loading rate of 25 mm/s are presented in Fig. 1 in the form of the stress-strain diagrams. By analyzing the data of these experiments, we can make the following conclusions:

(a) the investigated rigid films are characterized by the absence of the segment of yield: after elastic deformation, the material immediately passes to the stage of linear hardening and stays there up to failure, which explains its high strength and load-carrying ability;

(b) for strains lower than 4–5%, we observe geometrically nonlinear deformation (characterized by relatively large displacements and a linear dependence of the level of stresses σ on the level of strains ε);

(c) with an insignificant error, the material of the film can be regarded as isotropic; Poisson’s ratios of the specimens cut out along the length of the roll and in the transverse direction vary insignificantly (μ = 0.22–0.25 and 0.21–0.23, respectively);

(d) in our experiments, the levels of fracture strains and the corresponding stresses varied within the ranges ε = 45.7–75.6% and σ = 58.51–81.94 N/mm².

* These experiments were carried out by A. A. Sikora.
Fig. 1. Dependences of the tensile force $P$ on the absolute elongation $\Delta l$ for specimens subjected to tension along (1) and across (2) the direction of the roll.

Fig. 2. Experimental OLC and schematic OAC stress-strain diagrams.

It is reasonable to separate polymeric materials used to produce films into a special class of rigid elastoplastic materials without a segment of yield but with linear hardening. A film is regarded as an elastic two-dimensional body with very small thickness, which may undergo finite displacements and which has infinitesimally small bending moments and shear forces.

To characterize rigid polymeric materials, we construct, according to [8], a generalized schematic $\sigma - \varepsilon$ diagram (Fig. 2). On the basis of this diagram, we can determine the limiting values of elastic strains $\varepsilon_e$ and the corresponding stresses, Young's modulus $E = \tan \varphi$, and the modulus of hardening $E_h = \tan \psi$. To plot this diagram, we draw lines $MC$ and $ON$ tangent to the actual diagram.

It follows from Fig. 2 that

$$
\sigma^* = E\varepsilon, \quad 0 \leq \varepsilon \leq \varepsilon_e,
$$

$$
\sigma^* - \sigma_e^* = E_h (\varepsilon - \varepsilon_h), \quad \varepsilon_e < \varepsilon \leq \varepsilon_f,
$$

where $\varepsilon_f$ is the level of fracture strains and the asterisk means that we consider a deformed configuration.

It is necessary to complement relations (1) by the relation of unloading, which is obvious.

Further, by using relations (1), we now construct the determining relations for the film. For this purpose, we suppose that the plane of the film is referred to the Lagrangian curvilinear orthogonal coordinates $\alpha_1, \alpha_2$ both in the deformed ($C_f$) and nondeformed ($C_0$) states. Assume that $\vec{r} = \vec{r}(\alpha_1, \alpha_2)$ and $\vec{r}^* = \vec{r}(\alpha_1, \alpha_2) + \vec{u}(\alpha_1, \alpha_2, t)$ are, respectively, the radius vectors of points of the film in the nondeformed and deformed states, $\vec{u}$ is the vector of displacements, $t$ is time, and $ds^2 = d\vec{r} \cdot d\vec{r}$, $ds^* = d\vec{r}^* \cdot d\vec{r}^*$ are the squares of infinitesimal distances between arbitrary points of the film. Then the measures of the deformed state of the film $\varepsilon_1, \varepsilon_2$, and $\varepsilon_{12}$ are determined according to the Cauchy–Green approach: