OPTICS AND SPECTROSCOPY

MECHANISM OF SHAPE FORMATION OF OPTICAL STRUCTURES IN A NONLINEAR FIZEAU INTERFEROMETER AS THE MIRROR IS SHIFTED AND THE BEAM DIMENSIONS ARE VARIED

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The article is concerned with the study of shape formation in a nonlinear Fizeau interferometer with two-dimensional feedback. In a preceding study [1], an approach was presented by means of which it is possible to elucidate the mechanisms underlying the formation of the simplest optical structures for particular values of the parameters of the interferometer and the input optical beam. In the present article, an attempt is undertaken to account for the production of shear instability type structures and structures that arise as the beam dimensions in the feedback loop are varied.

In the present article mechanisms underlying the creation of shear instability type structures and structures that arise as the dimensions of a laser beam in the feedback loop are varied are discussed. A phenomenological approach previously proposed by the present authors, the scope of which was evaluated in [1], will be used. A model of the process of shape formation is provided by the dynamic equation of the phase advance \( u(x, y, t) \) from [2], which is represented in the following form:

\[
\frac{\partial u(x, y, t)}{\partial t} = D \frac{\partial}{\partial x} u(x, y, t) - \frac{u(x, y, t)}{\tau} + \frac{Z \cdot n_z I(x, y, t)}{\tau},
\]

\[
I(x, y, t) = (1-R) I_{in} \left[ 1 + \gamma \cdot \cos[u(x, y, t)] + \beta \right].
\]

From physical experiments [2], it is known that a shear instability type structure is created as the interferometer mirror is shifted a distance \( \delta \neq 0 \) in the feedback loop (Fig. 6.20 from [2]). To account for the conditions that arise as this type of structure is created, we will suppose that the shift has occurred along the \( x \)-axis and that its magnitude is a positive quantity \( dx \). Then the beam’s plane of cross-section may be partitioned into \( N \) regions, where the number \( N \) is equal to int(2/dx) + 1. From the physics of processes in an interferometer, it is known that the electro-optical effect in a nonlinear medium is maintained by interference of two beams, the input beam and the beam that has passed through the feedback loop [2]. Since the beam in the feedback loop has experienced displacement, only its central portion experiences interference with the input beam (the region from \( S_2 \) to \( S_N - 1 \)). Following its transit across the feedback loop, the peripheral (i.e., relative to the \( OX \) axis) portion of the beam escapes from the resonator and does not participate in the formation of the structure (this is the region \( S_N \)). It is our hypothesis that the dynamics of the process leading to the creation of the structure is specified by that portion of the input beam (relative to the \( OX \) axis) which the beam that has passed through the feedback loop (i.e., the region \( S_1 \)) does not interfere with (due to the displacement). Let us show that this is indeed the case.

Since the region \( S_1 \) does not participate in the interference process, the phase advance in this region, \( u(S_1, t) \), is determined not only by relaxation and diffusion, but also by the nonlinear electro-optical effect, which is maintained solely by...
Fig. 1. Dependence of steady-state solutions \( u(S_m, t) \) on the parameter of nonlinearity \( K \) with \( D = 0, \gamma = 1 \).

Fig. 2. Dependence of phase advance on coordinate along the \( OX \) axis with \( u(x, y, 0) = 1; \delta = 0.2; K = \pi/2; D = 1 \text{ sec}^{-1}; \tau = 0.001 \text{ sec}; \) and \( \gamma = 1 \).

the input beam. Since the intensity \( I_{in} \) is a constant quantity, once the rates of the oppositely directed phase variations (i.e., that due to relaxation, diffusion, and the electro-optical effect) have been made comparable, there are no further variations in the phase advance in the region \( S_1 \). Then the intensity of the interference field \( I(S_2, t) \) in a neighboring region \( U(S_1, t) \) is also invariant. Consequently, the phase advance \( u(S_2, t) \) is also invariant. Reasoning by analogy, we may extrapolate the conclusion regarding the invariance of the intensity of the interference field to the regions \( S_3, \ldots, S_N \), which means that the phase advance is constant in these regions. However, the fact that the intensity of the interference fields in the regions \( S_1, \ldots, S_N \) is constant does not mean that their intensities are all the same.

To verify this fact, let us first consider the process of shape formation, ignoring the influence of diffusion. Then, if it is assumed that phase advance is constant, i.e., \( du(x, y, t)/dt = 0 \), its magnitude will be determined, according to \( (1) \), by the value of the intensity. Consequently, in region \( S_1 \) the phase advance will increase linearly with increasing \( I_{in} \). Let us see how this occurs in the other regions as \( I_{in} \) increases. Because of the properties of the cosine function, which occurs in Eq. \( (2) \), in the range of values of \( u(S_1, t) \) from 0 to \( \pi/2 \), it turns out that \( I(S_2, t) > I(S_1, t) = I_{in} \). Consequently, values of \( I_{in} \) are possible such that \( u(S_2, t) > u(S_1, t) > \pi/2 \), but that means that \( I(S_3, t) < I(S_1, t) = I_{in} \). Thus, there exists a range of values of \( I_{in} \) for which the condition \( u(S_3, t) < u(S_1, t) < u(S_2, t) \) is fulfilled. Reasoning by analogy, it may be proved that over this range of values, all \( u(S_m, t) > u(S_1, t) \), where \( m = 2, 4, \ldots \), while \( u(S_m, t) < u(S_1, t) \), where \( m = 3, 5, \ldots \) (Fig. 1). This means that a structure is created in the plane of cross-section in which regions with phase advance greater and less than \( u(S_1, t) \) alternate.

Let us now turn our attention to the influence of diffusion. It is clear that if there is a phase discontinuity at the boundary between the regions, the diffusion processes will smooth it, and if the magnitude of the shift \( dx \) that determines the dimension of the region \( S_m \) is small, the variations in the magnitude of \( u(x, y, t) \) will become indistinguishable from one region to the next. But the influence of diffusion is even more clearcut when the value of \( I_{in} \) is such that \( u(S_1, t) = \pi/2 \). This situation is distinguished by the fact that a beam that has passed through the feedback loop is extinguished by the input beam, as a result of which the phase advance \( u(S_2, t) \) is determined by the quantity \( I_{in} \). That is, ignoring local interactions \( (D = 0) \), the advance \( u(S_2, t) \) is also equal to \( \pi/2 \). Obviously, such a conclusion is also valid for the other regions. If \( D \neq 0 \), the effect of diffusion leads to nonstationarity, and, thus, the phase advance \( u(S_1, t) \) differs from \( \pi/2 \). Next, an interference field \( K(S_3, t) \neq I_{in} \) is formed in \( S_3 \), increasing the difference between \( u(S_3, t) \) and \( \pi/2 \), with the magnitude of this difference growing constantly from one region to the next. Therefore, a shear instability type structure has the form shown in Fig. 2, which depicts the section \( u(x, y, t) \) along the \( OX \) axis.

If \( I_{in} \) now specifies a range of values of \( u(S_1, t) \) from \( \pi/2 \) to \( \pi \), successive alternation of the maxima and minima of the phase advance is reversed, and, in this form, is retained only within some range. The reversal of the order of alternation of the extrema is due to a change over to inverse phase relations between the interference beams. The breakdown in the process of alternation is caused by the fact that the intensity of the interference fields in the regions \( S_m \) may not only produce phase