We consider the stochastic dynamics that is the Boltzmann-Grad limit of the Hamiltonian dynamics of a system of hard spheres. A new concept of averages over states of stochastic systems is introduced, in which the contribution of the hypersurfaces on which stochastic point particles interact is taken into account. We give a rigorous derivation of the infinitesimal operators of the semigroups of evolution operators.

Introduction. The stochastic dynamics that corresponds to the Boltzmann hierarchy was recently proposed in papers [1, 2].

In the present paper, we prove that the stochastic dynamics is a certain limit of averages over the sphere of the Hamiltonian dynamics of system of hard spheres as their diameter tends to zero (the Boltzmann-Grad limit). We define the domain (set) of interaction in which the stochastic dynamics differs from the Hamiltonian dynamics of the free particles.

By using the concept of the domain of interaction, we define the operator of evolution for the stochastic dynamics and its infinitesimal operator. We prove that the operator of evolution of stochastic particles and its infinitesimal operator are the limits of the averages of the operator of evolution of a system of hard spheres and its infinitesimal operator, respectively, over the sphere as its diameter tends to zero.

Thus, in this paper, we present the rigorous derivation of the new concept of the stochastic dynamics of a system of point-particles as the limit of the average over the sphere of the Hamiltonian dynamics of system of hard spheres as its diameter tends to zero.

The crucial circumstance in this new concept of the stochastic dynamics and associated with it an average of observables over states of system of point-particles, that are governed by the stochastic dynamics, consists in taking into account the hypersurfaces of lower dimension than the phase space. In the traditional statistical mechanics hypersurfaces of lower dimension are neglected. In the next publications we will show that in solutions of the Boltzmann equation and the Boltzmann hierarchy the same hypersurfaces of lower dimension are taken into account. Namely, the terms of the series of iterations for the Boltzmann equation, as well as for the Boltzmann hierarchy, can be represented through the integrals over the hypersurfaces of lower dimension on which the stochastic particles interact.

Thus, the new concept of the stochastic dynamics corresponds to the Boltzmann equation and the Boltzmann hierarchy.

1. Stochastic trajectories as the limit of the Hamiltonian trajectories of hard spheres as diameter tends to zero. First, consider two hard spheres with diameter \( a \).
and mass 1. Denote by \((q_1, p_1) = x_1, (q_2, p_2) = x_2\) the positions of their centers and their momenta; \(x_1\) and \(x_2\) are their phase points.

We fix the initial momenta \(p_1, p_2\) and consider the position \(q_1^0, q_2^0\), such that the vector \(q_1^0 - q_2^0\) is parallel to the vector \(p_1 - p_2\) and \((p_1 - p_2) \cdot (q_1^0 - q_2^0) < 0\). Then for given \(q_2^0\) consider the semisphere \(q_2^0 - a \eta, \, \eta \in S^2, \quad \eta \cdot (p_1 - p_2) < 0\). As the initial position of the first sphere, we take the point \(q_1^0\) and, as the initial positions of the second sphere, we take the points \(q_2^0 - a \eta, \, \eta \in S^2\), (Fig. 1).

![Figure 1](image)

We consider a positive (increasing time) \(t \geq 0\) and \(t = 0\) is initial time. It is obvious that, at the time

\[
\tau = \frac{|q_2^0 - q_2^0|}{|p_1 - p_2|},
\]

the particles collide and touch each other at the point \(q_1^0 - \frac{a}{2} \eta + p_1 \tau\). After the elastic collision, their momenta are

\[
p_1^* = p_1 - \eta \eta \cdot (p_1 - p_2),
\]

\[
p_2^* = p_2 - \eta \eta \cdot (p_1 - p_2), \quad \eta \in S^2.
\]

The corresponding Hamiltonian trajectory is defined as follows:

\[
\begin{align*}
Q_1(t) &= q_1 + p_1 t, \quad P_1(t) = p_1, \quad Q_2(t) = q_2 + p_2 t, \quad P_2(t) = p_2, \quad t \leq \tau, \\
Q_1(t) &= q_1 + p_1 \tau + p_1^*(t - \tau), \quad P_1(t) = p_1^*, \\
Q_2(t) &= q_2 + p_2 \tau + p_2^*(t - \tau), \quad P_2(t) = p_2^*, \quad t > \tau,
\end{align*}
\]

for all \(q_2 = q_2^0 - a \eta, \, \eta \in S^2, \) and fixed \(q_1 = q_1^0\). Denote by

\[X^d(t) = (Q_1(t), P_1(t), Q_2(t), P_2(t))\]

the trajectories (1.3) of two hard spheres.

We defined by (1.3) the bunch of trajectories which is characterized by the vector \(\eta \in S^2\). Now let the diameter \(a\) tends to zero. Then for \(t \leq \tau\), the entire bunch of trajectories (1.3) is shrunk to the single trajectory

\[
\begin{align*}
Q_1(t) &= q_1^0 + p_1 t, \quad P_1(t) = p_1, \\
Q_2(t) &= q_2^0 + p_2 t, \quad P_2(t) = p_2, \quad t \leq \tau.
\end{align*}
\]