THE INFLUENCE OF STATISTICAL ERROR
IN RADIOACTIVITY COUNTING ON THE PRECISION
OF ISOTOPE DILUTION ANALYSIS, I

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(Received June 30, 1971)

The influence of the statistical error in measuring radioactive samples in direct isotope dilution analysis is analytically studied for several practically important sets of conditions. Formulas are given to calculate the optimum amount of radioactive substance to be added in order to minimize the error involved. Both cases with negligible background count rates and cases where the background must be considered are treated. In some cases recommendations are given for the times of measurement of background, undiluted and diluted sample.

Introduction

Some aspects of the title problem have been studied by several authors.¹ A literature review and the results of the so far most complete investigation made by one of us (M. K.) were reported in a recent monograph.² In this paper we want to give readily usable solutions of the problem for several useful sets of conditions so far ignored, and supply additional data for sets of conditions which have been preliminarily treated. In the following parts of this series several special cases will be treated.

The practical situation encountered by a user of isotope dilution analysis (IDA) is usually as follows. The specific activity of the isotope available (So) is known and cannot be increased, while the unknown amount (Wₓ) is known approximately, or can be determined approximately by preliminary experiments. We want to know the desired amount (Wₒ) of the radioactive substance to be added so that the final relative error of the analysis (ΔWₒ/Wₓ = ε) is as low as possible. Furthermore, we are interested in knowing the value of ε which can be expected in our case. It will be shown later, but can be understood here intuitively, that the desired amount Wₒ depends not only on So and Wₓ but generally on several other conditions such as the times of measurement of the undiluted sample (tₒ), diluted sample (t₂) and background (tₐ); on the dependence of the yield of isolation upon the total amount of the substance (Wₓ + Wₒ) in the purification and isolation steps; and on the way of preparing the sample for measuring So. If all these *other con-
ditions’ are clearly indicated, no matter what they may be, the desired value \( W_0 \) can always — at least in principle — be calculated, together with the error \( \varepsilon \) involved. Our task is to define several of these possible ‘sets of conditions’ and give formulas, tables or graphs which will enable the user of IDA to find \( W_0 \) and \( \varepsilon \) easily, rapidly and without many additional calculations. In choosing the ‘sets of conditions’, two criteria were considered: first, only such sets of conditions were treated which have a sufficiently high probability of occurring in real problems of IDA; second, the sets of conditions which lead to very complicated formulas requiring lengthy and tedious calculations by the analyst using IDA were ignored. The different sets of conditions studied and elaborated below apparently allow the handling of the overwhelming majority of the practical cases of direct IDA with radioactive isotopes.

**Basic formulas and assumptions**

All the basic formulas underlying the subsequent calculations are well known, but it is desirable to give them here for the sake of completeness.

The basic formula of IDA is

\[
W_x = W_0 \left( \frac{S_0}{S_2} - 1 \right) = W_0 \left( \frac{R_t - b}{W_1} \cdot \frac{W_{is_2}}{R_t - b} - 1 \right) = W_0 \left( \frac{R_0}{W_1} \cdot \frac{W_{is_2}}{R_2} - 1 \right)
\]

(1)

where 
- \( S_2 \) — specific activity after dilution,
- \( R_{t0} \) — count rate (including the background count rate \( b \)) of the sample containing \( W_1 \) grams of the undiluted radioisotope,
- \( R_{t2} \) — count rate (including background) of the diluted sample containing \( W_{is_2} \) grams of diluted radioisotope.

The values \( R_0 \) and \( R_2 \) are the count rates corrected for background.

The use of the well-known Gaussian formula\(^3\) of error propagation is essential in our calculations. This formula states that if \( E \) is a function of measurable values \( x, y, z, \) etc., then the average statistical error \( \Delta E \) is given by

\[
\Delta E = \pm \sqrt{\left( \frac{\partial E}{\partial x} \right)^2 + \left( \frac{\partial E}{\partial y} \right)^2 + \left( \frac{\partial E}{\partial z} \right)^2 + \ldots}
\]

(2)

where \( \partial E/\partial x \), etc. are obtained by partial differentiation, and \( \Delta x \), etc. are average statistical errors of the measurements of the values \( x, \) etc.

If the number of counts measured during a certain period of time is \( N \), then (as follows from the statistical theory of radioactive decay) the standard deviation is \( \Delta N = \pm \sqrt{N} \). From this, we immediately obtain the standard deviation of a count rate

\[
R_t = \pm \sqrt{N/t} = \pm \sqrt{R_0/t} = \pm \sqrt{R_2/t}
\]

if we measure during time \( t \).