APPLICATION OF COLLOCATION FOR THE PLANNING
OF GRAVITY SURVEYS

Abstract

Least squares collocation can be used to determine the density of a gravity survey, when the object of the survey is:

1. To produce a (free-air) gravity anomaly map, so that point gravity anomaly values can be interpolated with a standard error of $\pm X_1$ mgal,

2. To interpolate deflections of the vertical with a standard error of $\pm X_2$ arc. sec. between astronomical stations $Z$ km apart,

3. To compute an upward continuation of a point gravity anomaly to a height of $Z$ meters with a standard error of $\pm X_3$ mgal,

4. To compute mean gravity anomalies of block size $Z$ degrees with a standard error of $\pm X_4$ mgal,

5. To obtain a (local) gravimetric geoid with a standard error of $\pm X_5$ meters,

6. To compute density anomalies at a depth of $Z$ km with a standard error of $\pm X_6$ g/cm$^3$.

1. Introduction

A gravity survey of a region may be made with the purpose of mapping the gravity variation and/or estimating one or more geophysical quantities from the observed gravity values.

In both cases, the survey must be planned so that the quantities of interest can be estimated with a specified standard deviation. In the first case, where we want to map the gravity variation, we may specify the density of the survey e.g., by requiring that free-air gravity anomalies can be interpolated everywhere within the region with a standard deviation of $\pm X_1$ mgal.
An estimate of the interpolation error may be determined for the linear least squares prediction method, Heiskanen and Moritz (1967, Section 7-6) by a simple equation, Ibid (1967, eq. (7-64)):

\[ m_p^2 = c_{pp} - \{ c_{pi} \}^T \{ c_{ij} \}^{-1} \{ c_{pj} \}, \tag{1} \]

where \( m_p^2 \) is the standard error of least squares prediction, \( c_{pp} \) is the mean square variation of the gravity anomalies, \( C = \{ c_{ij} \} \) is a \( q \times q \) matrix of covariances between the observed gravity anomalies and \( C_p = \{ c_{pi} \} \) is the \( q \) vector of covariances between the anomaly to be predicted and the observed quantities.

The equation (1) has been used to estimate the standard error of e.g. potential coefficients, height anomalies and deflections of the vertical by simply propagating the error through the equations relating these quantities to the gravity anomalies, namely the equations of Stokes and Vening-Meinesz, see e.g. Groten and Moritz (1964) and Heiskanen and Moritz (1967, sections 7-7 and 7-9).

This type of procedure has some disadvantages. Firstly, several integrals (with singular kernels) will have to be evaluated. Secondly, the procedure cannot be applied directly when data of a different kind (e.g. deflections and gravity anomalies) are used together.

However, the method of least squares collocation will furnish us with estimates of the standard error of prediction. These estimates are computed by an expression similar to (1). The difference is, that the covariances are not between gravity anomalies anymore, but between the quantities used for the estimation (the observations) and the quantity to be estimated.

In Section 2, we will present briefly the theory of least squares collocation as applied in physical geodesy and discuss the interpretation of the estimate of the error of prediction, furnished by this method.

We have applied least squares collocation to a set of point gravity anomalies using different covariance functions, and predicted other anomalies and deflections of the vertical. The computational procedure used is described and a comparison of the actual and the estimated error of prediction is presented in Section 3.

In the following Section 4, the results of some test computations are presented. Eight sets of fictitious point gravity anomalies spaced with variable density in a \( 1^\circ \times 1^\circ \) block have been used to compute estimates of the error of prediction for point or mean gravity anomalies, height anomalies and deflections of the vertical within the area.

In the final section we explain how the error estimates, obtained by the method of least squares collocation, can be used in the planning of a gravity survey.