METHOD OF SIMULTANEOUS DETERMINATION OF ASTRONOMIC LATITUDE, AZIMUTH AND LONGITUDE BY OBSERVING ONLY TIME AND HORIZONTAL ANGLES BETWEEN A PAIR OF STARS NEAR ELONGATION AND A SOUTH STAR NEAR MERIDIAN

Summary

This paper endeavours to evolve a method of simultaneous determination of astronomic latitude, azimuth and longitude from observations of a star-pair near their times of east and west elongations and a south star near its time of meridian transit. The star-pair of observation being within a short distance of elongation, either east or west, their motion in azimuth then is extremely slow and the small error in time has therefore insignificant effect on their azimuth, and in addition, the south star with its azimuth known from observations of the previous star-pair, being very fast-moving, the method is especially suitable for determining the astronomic latitude, azimuth and longitude by observing only the time and horizontal angles between them, and also a reference mark for ensuring the orientation of the horizontal circle side by side.

1. General Principle of the Method

As the star pair of observation are near elongation, either east or west, the position is especially favourable for observing the horizontal angles, the motion in azimuth then being extremely slow (Doolittle, 1895). Moreover, from the differential relation:

$$dA = \sin A \cdot \tan h \cdot d\phi + \cos \phi (\tan \phi - \cos A \cdot \tan h) \cdot d\lambda$$

where \(\tan \phi = \cos A \cdot \tan h\) for stars observed near elongation and \(\sin A \cdot \tan h = 0\) for stars observed near meridian transit, it is apparent that although errors in time or longitude in the case of stars near elongation will have an insignificant effect on the determination of their azimuths, the latter will, however, be greatly affected by the error in the assumed latitude. Therefore, the difference between the measured angle between an east and a west star and that computed from their azimuths using the assumed latitude \(\phi_0\), will be due practically to an error \(d\phi\) in the assumed latitude only, thereby reducing the correction to the assumed latitude, to the form (Bhattacharji, 1959, Mueller, 1969):

$$\phi - \phi_0 = -d\phi = \frac{dH}{\sin A_{OE} \cdot \tan h_{OE} - \sin A_{OW} \cdot \tan h_{OW}}$$

in which \( dH = (AO_W - AO_E) - (H_W - H_E) \); the quantity \( (AO_W - AO_E) \) is the computed horizontal angle between the two stars near east and west elongations, that is, the difference of their azimuths as computed using the assumed latitude \( \phi_0 \); the quantity \( (H_W - H_E) \) corrected for collimation, diurnal aberration and dislevelment errors of the telescope, represents the measured value of the same, i.e., the difference of their horizontal circle readings observed. Having obtained the correction to the assumed latitude as above, the computed azimuths of the stars observed near elongation, can be straight away corrected for the error in the assumed latitude separately, and individually added to the difference: corrected horizontal circle reading to the reference mark (R.M.) minus that to the individual star in order to obtain the required azimuth of the reference mark. Now, on having determined the azimuth of the reference mark as above, the corresponding azimuth of the south star also can be deduced straight away simply by adding to it the quantity \( (H_S - H_R) \) representing the measured angle between the reference mark and the south star after having corrected for collimation, diurnal aberration and dislevelment errors as before, and since \( \sin A \cdot \tan h \cdot \phi = 0 \) for stars near meridian transits, any difference between this and the corresponding quantity \( (A_O_S - A_R) \) representing the difference between their azimuths, \( A_O_S \) being computed using the assumed longitude \( \lambda_0 \), has to be due practically to an error \( d\lambda \) in the assumed longitude only, thereby reducing the longitude correction to the form (Mueller, 1969, Bhattacharji, 1958):

\[
\lambda - \lambda_0 = - d\lambda = \frac{- dH}{\cos \delta_S \cdot \cosec (\phi - \delta_S)}
\]

in which \( dH = (A_O_S - A_R) - (H_S - H_R) \)

and \( \cos \phi \left( \tan \phi - \cos A \cdot \tan h \right) = \cos \delta_S \cdot \cosec (\phi - \delta_S) \) for a south star near transit.

Hence the final values of astronomic latitude \( \phi \), azimuth \( A_R \) and longitude determined as above, are:

\[
\phi = \phi_0 - d\phi
\]

\[
A_R = (A_O_E - \sin A_O_E \cdot \tan h_O_E \cdot d\phi) + H_R - H_E = A_E + H_R - H_E
\]

Also \( A_R = (A_O_W - \sin A_O_W \cdot \tan h_O_W \cdot d\phi) + H_R - H_W = A_W + H_R - H_W \)

or \( A_R = H_R + \frac{1}{2} \left[ (A_E - H_E) + (A_W - H_W) \right] \),

where \( A_E = A_O_E - \sin A_O_E \cdot \tan h_O_E \cdot d\phi \);

\[
A_W = A_O_W - \sin A_O_W \cdot \tan h_O_W \cdot d\phi
\]

and \( \lambda = \lambda_0 - d\lambda \)

2. Selection of Stars

Three stars having their magnitudes varying from 3.0 to 6.5 and depending on the precision of theodolites in use, of which the two are within 10 to 15 minutes of their times of elongation, east and west, and the third within about the same time of its meridian transit, south of the zenith, are selected as an observation unit. Two such units are needed for the second-order and four units for the first-order determinations, and