SOLUTION OF THE GEODETIC BOUNDARY VALUE PROBLEM

Summary

The possibility of improving the convergence of Molodensky's series is considered. Then new formulas are derived for the solution of the geodetic boundary value problem. One of them presents an expression for the boundary condition which involves a linear combination of Stokes' constants and surface gravity anomalies. This differs from the usually used relation by the appearance of additional terms dependent on second order terms with respect to the elevations of the earth's surface. The formulas are derived for Stokes' constants and the anomalous potential in terms of surface anomalies. As compared to the Taylor's series of Molodensky, they are presented in the form of surface harmonic series. Due regard is made to the earth's oblateness, in addition to the terrain topography.

1. Introduction

The problem of determination of the earth's gravitational field on the basis of given surface anomalies can be attacked in two different ways. The so-called continuous approach was first developed in the most general form by Molodensky (Molodensky et al., 1960). In his theory the anomalous potential \( T \) is expanded in a power series with respect to the auxiliary shrinking parameter \( k \). At every step of approximation the solution for \( T \) is presented in a form of surface integrals and therefore the gravity anomalies have to be known all over the earth's surface. The other way to find the external gravitational field lies in determination of Stokes' constants \( C \) and \( S \) by statistical methods from the boundary condition of the form \( f(C, S) = \Delta g \) which relates a linear combination \( f(C, S) \) of Stokes' constants to the surface anomalies \( \Delta g \). The function \( f(C, S) \) depends on the spherical coordinates \( \varphi \) and \( \lambda \) of a given point and is presented as a spherical harmonic series, \( C \) and \( S \) being its coefficients. This problem is known as the discrete boundary value problem (Bjerhammar, 1975) since Stokes' constants are assumed to satisfy the given finite set of values of \( \Delta g \).

In the present paper both approaches are considered. We start from the analysis of the possibility to improve the convergence of Molodensky's expansion of the anomalous potential. Strictly speaking, Taylor's series similar to Molodensky's one can not be convergent since the earth's potential is not an analytical function along the radius--vector of the earth on and under its surface. As a matter of fact, due to non—homogeneous structure of the earth the second derivative of the potential breaks on the surface and thus even the smoothing of the physical surface (small values of the shrinking
parameter $k$) is not likely to secure the convergence. But Taylor's series can be used for sure as an asymptotic expansion. It will be shown now that a slight modification of Molodensky's series can provide an expression which is closer to the real potential, at least at the first steps of approximation.

Taylor's series are not the only possible way to expand the potential $T$. In particular this function can be presented as a convergent series in surface harmonics (Petrovskaya, 1977). This expansion differs from the well-known Laplace's series in satellite theories by the appearance of correction terms in the space $\Omega$ between the earth's surface and the sphere inclosing all the earth's masses. The corrections are functions of the earth's shape and the density distribution. In the present paper they will be expressed in terms of the anomalies $\Delta g$ and the heights $h$. This solution differs from Molodensky's one starting with the second order terms with respect to $h$.

The boundary relation $f(C, S) = \Delta g$ mentioned above is revised too. It suffers from not being well-grounded. This is quite rigorous indeed for the spherical surface of a body rather than for the physical surface of the earth. Now a new expression of the boundary condition will be provided of the form $f(C, S) = \Delta g + F$ where the function $F$ presents correction terms to the infinite series in the left-hand side of this equality. The corrections appear in the same space $\Omega$ including the surface, as in the previous case. The function $F$ will be presented in the form of surface integrals dependent on $\Delta g$ and $h$. It can be expressed in terms of Stokes' constants as well. The new boundary condition can be used for Stokes' constants determination on the basis of given values of the surface anomalies $\Delta g$ in the same way as the previous one was applied. If the function $F$ is expressed in terms of Stokes' constants then the formula $f(C, S) - F(C, S) = \Delta g$ performs downward continuation, that is the surface values of $\Delta g$ can be estimated starting from the values of Stokes' constants received from satellite observations.

All the relations derived are based on one and the same Brovar's integral equation (Brovar, 1963) for the determination of the generalized density of a surface layer.

2. Remarks on Molodensky's series

The solution of Molodensky's problem is presented as a surface layer potential

$$T = R^2 \int \int \frac{\chi}{r^2} d\omega$$

Here $\chi$ is the surface layer density factored by $r^2 R^{-2} \sec \beta$, $r^2 = r^2 + r_1^2 - 2 r r_1 \cos \psi$, $r$ and $r_1$ are the radii—vectors of the points $P$ and $P_1$ on the earth's surface, $\psi$ is the angle between them, $\omega$ means the unit sphere, $d\omega$ is a solid angle element and $\beta$ is the inclination of the earth's surface. The subscript one refers to the variable point on the earth's surface. The function $\chi(\varphi, \lambda)$ is sought for as a solution of the integral equation which is derived by inserting (1) into the boundary condition

$$\lim_{r \to \bar{r}} \left( \frac{\partial T}{\partial r} + \frac{2}{r} T \right) = -\Delta g$$

$$r \to \bar{r}$$