COLLOCATION AND LEAST SQUARES METHODS AS A TOOL FOR HANDLING GRAVITY FIELD DEPENDENT DATA OBTAINED THROUGH SPACE RESEARCH TECHNIQUES *

Abstract

Least squares adjustment and collocation methods have in the last decade been the tool for extracting gravity field information from data obtained through space research techniques (satellite orbit tracking, altimeter observations, doppler determined positions), and when combining these data with data observed at the surface of the Earth.

The mathematical framework for the two models is described and the models are compared. It is shown that the two methods only become equivalent in cases where the number of parameters are equal to the number of observations.

It is pointed out that several arbitrary choices (of parameters, weights and norms) will have to be made before the methods can be applied, and that further investigations are needed in order to justify the specific choices.

1. Introduction

A knowledge of the gravity potential of the Earth is required in order to solve problems in navigation, geodesy, oceanography and solid earth physics (see Mather (1974–1977), Holland et al., (1976)).

Earlier it was envisaged that the gravity potential could be determined as a solution to a certain boundary value problem for the Laplace operator. The boundary values were the gravity anomalies, for which a global coverage were required.

But also other quantities give information about the gravity potential, namely astronomical latitude and longitude (which give the direction of the gravity vector) and the gravity gradients. Further types of quantities became available by the advent of the artificial satellites. These quantities were (and is) obtained by satellite tracking or by remote sensing techniques. Satellites equipped with gradiometers will make further types of observations available. And we should not forget, that geopotential heights, obtained by levelling, in combination with cartesian coordinates (determined e.g. by doppler techniques) in itself is information about the gravity potential.

It is the purpose of this paper to discuss two currently applied methods for the handling of this heterogeneous dataset. In Section 2 we describe the basic characteristics of the mathematical framework within which the gravity potential can be considered. We discuss in Section 3 least squares adjustment methods, in Section 4 the minimum norm


collocation method and in Section 5 the least squares collocation method. This method is a collocation method where the norm has been selected so that a least square condition is fulfilled. Finally in Section 6 we describe some of the applications of the methods and discuss some of the problems associated with the application of the methods.

The paper does not contain any new results. However, a number of papers published during the last years have revealed that the relationship between the collocation and the adjustment methods was not fully understood. I have therefore thought it timely to try to synthesise the present viewpoints, combined with a description of the use of the two methods in an area very important to geodesy.

It has only been possible to make this "synthesis", because I had the opportunity to participate in some clarifying discussions which took place at the recent Ramsau Summerschool (2' International Summerschool in the Mountains, Ramsau, Austria, 23 August — 2 September, 1978). Thanks to all (lecturers and students) who took part in these discussions.

2. The Mathematical Model for the Gravity Field

The gravity potential, \( W \), is the sum of the gravitational potential, \( V \), and the rotational potential. Let us suppose that we have adopted a certain reference potential, \( U \), which rotational part is equal to that of \( W \) and which contains the influence of all masses outside the surface of the Earth (the atmosphere, the moon, the sun, etc.). Then \( T = W - U \) is called the anomalous potential and it is a harmonic function outside the surface of the Earth. Let us denote the set of harmonicity by \( \Omega \). Points in \( \Omega \) (or in \( \mathbb{R}^3 \)) will be denoted \( P, Q \), with spherical coordinates \((\varphi, \lambda, r)\), \((\varphi', \lambda', r')\), respectively. \((\varphi = \text{geocentric latitude}, \lambda = \text{longitude} \text{ and } r = \text{distance from the origin})\).

The anomalous potential will be an element of a linear vector space of functions harmonic in \( \Omega \), \( H(\Omega) \). The vector space may be equipped with inner products, \(( , )\), which will have corresponding norms, \( \| \cdot \| \). (We will distinguish between different inner products or norms by a subscript). The elements of \( H(\Omega) \) for which a norm is finite will form a so-called Hilbert space. The spaces will be infinite dimensional, but separable, i.e. a countable basis exists. (We have to put some small restrictions on \( \Omega \) and not all norms will do).

The anomalous potential will be an element of such a space, for example with norm given by

\[
\| T \|_0^2 = \frac{1}{4\pi} \int_\Omega T(P)^2 \, d\Omega \quad (2.1a)
\]

or

\[
\| T \|_1^2 = \frac{1}{4\pi} \int_\Omega (T(P)^2 + (\nabla T)^2) \, d\Omega , \quad (2.1b)
\]

where \( \nabla T \) is the gradient of \( T \).

In Hilbert spaces any element may be represented by its expansion with respect to a complete, orthonormal system. Had \( \Omega \) been the set outside a sphere with radius \( R \) we would have